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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRACTICAL RIVER LABORATORY HYDRAULICS

Discussion

By Herbert D. Vogel, Assoc. M. Am. Soc. C. E.

Herbert D. Vogel,[41] Assoc. M. Am. Soc. C. E. (by letter)[41a].—Inasmuch as the main purpose in presenting a paper on practical river laboratory hydraulics was to draw out constructive thought on the subject, the many discussions which ensued have proved most gratifying. The fact that several have emanated from young men in the profession is also encouraging because, as the science itself is young, its full future development will rest upon engineers who have years of growth ahead of them. The principal point of disagreement among those offering discussions, fortunately, is one which should be easily settled by the newly appointed Special Committee of the Society on Hydraulic Research. No one individual can lay down symbols and nomenclature which will be universally acceptable, but an impartial committee, acting on the matter, should be able to arrive at a generally satisfactory terminology.

Professor Straub favors the designations used in the paper with the reservation that symbols for scale should represent whole numbers instead of proper fractions. This makes for a certain degree of simplification, but may cause confusion in the minds of those who have habitually expressed scales—as of maps—as proper fractions. The effects of distortion have been well brought out by Professor Straub and recent tests conducted at the U. S. Waterways Experiment Station have served to verify a number of his points. In the case of a model of the St. Clair River from which it was desired to obtain the hydraulic effects of submerged sills, it was found that a distortion of about 3 to 1 caused a sufficient departure from the Froudian law to give results materially different from those obtained in the case of a non-distorted model constructed subsequently. Changing the roughness of

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Note.—The paper by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., was published in November, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1934, by Messrs. Lorenz G. Straub, Paul W. Thompson, Ralph W. Powell, K. D. Nichols, and Frank W. Edwards; March, 1934, by Messrs. I. H. Patty, Charles S. Bennett, and Kenneth C. Reynolds; April, 1934, by Messrs. V. V. Tchikoff, Samuel Shulits, and Charles D. Curran; and May, 1934, by J. B. Egiazaroff, Assoc. M. Am. Soc. C. E.

[41] Lieut., Corps of Engrs., U. S. Army; Student, The Command and General Staff School, Fort Leavenworth, Kans.

[41a] Received by the Secretary August 24, 1934.

the distorted model served to bring the results into much closer agreement with those of the non-distorted model, but practicable limits of roughness could not be exceeded. Tests on the non-distorted model with discharges varying from Froudian relationships relatively the same degree as on the distorted model, were in close agreement with the results obtained on the latter, thus indicating that if Froudian relationships had been adhered to the distorted model would have given accurate results. In this case, the practicable limits of roughness having been reached, a decrease in slope throughout the model (to be obtained by progressively raising each template from head to foot of the model sufficiently to eliminate part of the increase in slope due to geometric distortion) was indicated as a necessity to comply with Froudian relationships.

Had the model been built for the determination of resulting bed configurations—that is, had it required a movable sand bed—an increased slope (additional to that imposed by distortion) or depth would have been necessary to develop adequate tractive force, and a greater departure from Froudian laws would have been necessary. Relatively few movable bed models comply with Froudian relationships, however, and if proper verification tests are conducted this departure is unimportant.[42] In a movable-bed model the principal objection to distortion is that angles of repose of sand banks are substantially the same as in Nature, while distortion requires that they be maintained much steeper. Professor Straub has explained this point to advantage.

Lieut. Thompson has introduced a new note in striking upon the Chezy coefficient as a criterion for controlling roughness in the model. This entire subject is one upon which much future research should be centered. At present, it can only be said that the sum of knowledge relative to roughness coefficients of natural rivers or models is pitifully small and that there is no index for the actual measurement of roughness. The symbols, n, m, or C, as commonly used, are factors introduced to make the answer "come out right", and include resistance of bends, loss in eddies, effects of changes of cross-section, unknown obstructions, such as wrecks of vessels, etc.

Professor Powell voices the attitude just expressed in stating the need for a "new and better formula for channel flow". His theoretical treatment of the problem from a basis of dimensional analysis is extremely interesting, although not necessarily conducive to aid in the practical design and operation of river models. As stated in his closing paragraph, however, the method is the principal contribution that can be advanced at this time, and any really satisfactory formula must follow from extensive tests on channels of varying characteristics. The American Committee of the World Power Conference, in 1932, undertook the task of co-ordinating research activities in this field.

The efforts of Lieut. Nichols and Mr. Patty to develop a suitable and dependable time scale are wholly laudable, but in spite of all present evidence to the contrary the writer still believes that, as was previously stated (see paragraph following Equation (12)),

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[42] See "Sand Mixtures and Sand Movement in Fluvial Models", by Hans Kramer, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., April, 1934, p. 443.

"There is no known 'short-cut' to such determinations, and formulas taking into account the transporting power of flowing water and the resistance to motion of grain particles, in suspension or otherwise, are almost invariably misleading."

Recognizing that for every condition of bank alignment or curvature there will result for each stage an ultimate bed development of unchanging nature, it seems more reasonable to assume a cycle of such duration as will fit into a normal working day, and then, for each condition to be tested, to repeat the cycle a sufficient number of times to insure the attainment of stable conditions. Both Mr. Patty and Lieut. Nichols recognize this fact, and the latter has made it particularly plain by his discussion that practical difficulties exist, which make an empirical formula nearly impossible of achievement.

The discussion by Mr. Edwards points out once more the possibility of reconciling the Froudian law with established laws of friction flow in open channels. This was tried in the case of the St. Clair River experiment, previously mentioned, and again in the canal-crossing experiment at Freeport, Tex. In the latter experiment the silt-transporting capacity of the model river had to be reconciled with the natural laws of hydraulic flow as established by Froudian principles. This was accomplished by designing the model so as to obtain a discharge scale based on the Manning formula, and, subsequently, roughening the concrete bed so as to obtain a value for Manning's n which would bring the discharge scale in perfect agreement with both the Froudian law and the Manning formula. A heavily stuccoed concrete bed produced the required frictional resistance ($n = 0.022$), and no other expedients were necessary to obtain compatibility. Another method of adjustment is to change the form of the relationship, $d = l^{\frac{3}{4}}$, or to add negative supplementary tilt to the model. Much experimental work must be done before laws can be established to meet the manifold ramifications of compatibility requirements. Eventually, research will point the way to true reconciliation of the requirements of Froudian laws, turbulence, and bed-load transportation, but for the present the first thing to be recognized is that, as a rule, each model is designed for a particular purpose—that is, to procure similarity of behavior with respect to a single type of phenomenon.

Mr. Bennett's discussion serves a valuable purpose in showing how model tests may be accomplished at very low cost when limited funds are available to the experimenter. The accuracy of experimental data is generally greater for models of large scale than for those of small dimensions, but frequently a model of modest proportions will serve to supply necessary information within the range of required accuracy. It would appear foolish to forego all model testing simply because the goal of achievement has been set too high. When funds are not available for extended research and experimentation, it is still possible that a less ambitious program will yield results sufficiently accurate for practical purposes.

Professor Reynolds joins with Professor Powell in protesting departure from the use of standard Greek symbols to designate viscosity, kinematic viscosity, and density. Since these symbols are not strictly a part of scale designations, it would seem wise to follow their suggestion and adhere a while longer to the more easily recognized notations.

Mr. Tchikoff offers a number of valuable ideas, all of which are interesting and few of which are subject to serious controversy. He expands somewhat on the statement in the paper (see heading, "Turbulence in Open-Channel Models") that: "For all practical purposes dynamic and kinematic similitude exists when the flow of water in the model is of the same character as the flow in Nature." He then raises the question, "How is one to judge when the two flows are of the same character?" This is answered by the presentation of a practical criterion, based upon the Bakhmeteff theory of hydraulic flow,[25] according to which the prototype and model must have the same kinetic flow factor and the discharges must be in proportion to the cubes of the hydraulic radii. As a basis for the foregoing criterion, it can be assumed that the character of silt transportation must be the same for the same silt in both channels. Mr. Tchikoff, however, does not make clear his definition of silt, or his reason for believing it necessary to use the same material in the model as in Nature. If by "silt" bed material is meant, the factors of slope and depth will control; hence, by judicious control of the model slope and by careful selection of the model sand, similarity of action is readily procured. It is desired to emphasize here, as elsewhere, that in general a model should be designed to fulfill a single specific mission; that the controlling factors should be carefully taken into account in each case; and that if more than a singe purpose is to be fulfilled a conscious check must be made to determine whether the original model—modified perhaps—will suffice, or whether an entirely new model should be constructed.

Mr. Shulits' discussion of the friction factor and its effects is rational and interesting. The method of simulating over-bank roughness by means of pebbles, has long been in use at the U. S. Waterways Experiment Station, but in recent months (1934) wire screens have been found more satisfactory, especially when it becomes necessary to represent floods of considerable height which would overtop small stones on the over-bank. Data are gradually being accumulated in all laboratories to determine values of the non-dimensional roughness coefficient, but the combinations of surfaces and types of models are so many as to make the problem a lengthy one. In the meantime, the situation is far from hopeless if adequate data are known of the river investigated. One method is to reproduce a water-surface profile in the model corresponding to less than bank-full flow in Nature; then, by comparing discharges in model and in Nature, to determine a definite discharge scale; next, using the discharge scale thus determined, to reproduce a flood of considerable known magnitude, for which gauge readings were taken in Nature; and, finally, to adjust the over-bank roughness on the model until a water-surface profile is procured which coincides with that of the natural river. Referring to Mr. Shulits' comments relative to the effects of temperature on the turbulence criterion, it is agreed that considerable importance should be attached to this item. In the general case, however, turbulence is not an all-important factor because if bed movement is a requirement there will always be high turbulence when slopes and depths are such as to result in the production of adequate tractive force.

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[25] "Hydraulics of Open Channels", by Boris A. Bakhmeteff, M. Am. Soc. C. E., p. 64.

Lieut. Curran states a timely warning in advising against a too optimistic interpretation of model results; but, nevertheless, recent progress has been such as to make quantitative answers frequently obtainable. In cases where exact reproductions of natural developments appear impossible, corrective factors may be applied to the model results with nearly complete satisfaction.

Professor Egiazaroff points out that, assuming turbulence to result whenever the product of velocity and depth exceeds 0.02, there remains little need for distortion of models, especially in the case of a movable bed. This would be true if turbulence were the only requirement, but, if the bed is of sand, then sand movement is another matter for consideration, and distortion may be necessary for attainment of the required tractive force. The rapid development of Russian laboratories and the excellent work being performed at Leningrad, make Professor Egiazaroff's remarks of particular interest, and his citation of recent experiments gives a valuable sidelight to his discussion.

During the past summer (1934) it has been the writer's privilege to revisit leading hydraulic ("flussbau") laboratories of Germany and, mentally, to compare their present installations and methods with those of 1929. While considerable progress has been made, particularly in the field of lock models, the advancement has been not nearly as rapid, or upon as broad a front, as in the United States. Probably the most important single contribution to the science is the work of Hans Kramer, Assoc. M. Am. Soc. C. E., on the movement of bed material in fluvial models.[43] Investigations supplementary to the studies of Captain Kramer are being carried on at present by the U. S. Waterways Experiment Station and at the Berlin Laboratory of the Prussian Government (Preussische Versuchsanstalt für Wasserbau, Schiffbau, und Erdbau) by Captain Hugh J. Casey, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., but the original work by itself has opened an entirely new field for model investigations. That this is true is evidenced by the fact that nearly every laboratory in the United States has recognized the importance of tractive force studies and is carrying the work forward along individual lines.

What the future will bring forth is difficult, if not impossible, to predict, but it seems certain that American engineers are coming more and more to appreciate the advantages following from tests of river and harbor models. In nearly every section of the country where important public works are being constructed, there can now be found the proposed works in miniature and under test by competent, well-trained, hydraulic-model engineers. It is to be hoped that the growth will be steady and sure, with a background of common sense and practical applicability constantly maintained, and that, as a result, model studies will take their proper place with other forms of engineering activities, not exaggerated in importance, not regarded as a cure-all for all troubles, but respected as an auxiliary method available to the practical engineer in connection with problems not readily susceptible to a purely theoretical analysis.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RAINFALL STUDIES FOR NEW YORK, N. Y.

Discussion

BY MESSRS. RHODES E. RULE, AND S. D. BLEICH

RHODES E. RULE[21], ASSOC. M. AM. SOC. C. E. (by letter)[21a].—The results of an analysis of rainfall records are presented in this paper, in which empirical formulas are derived and compared with the results of a parallel study by the probability method. The author finds that the "modified exponential method gives results more in accord with the actual data than the probability method." He adopted the "Type III" curve for use in his application of the probability method, but does not state why it was used except that it "fits the data best". Since goodness of fit was the sole criterion, it should be noted that a better fitting curve could have been obtained by (1) grouping the data; (2) fitting a separate curve to each group; and (3) taking the summation of all such curves to make the final frequency curve. Such a method would be cumbersome, and perhaps difficult to defend on rational grounds but, neverthless, it would be the best fitting curve. Carrying the argument a step further the data could have been plotted in the histogram form, and the fit would be perfect. Does this demonstrate that the data in their unsmoothed form are better than any theoretical smoothing of them?

Of course, this is not true and the fallacy of the argument lies in the fact that goodness of fit is not the sole test. In the final analysis the propriety of using a given type of curve must rest on a rational demonstration that the data follow some law which is exemplified by curves of the type adopted. Unfortunately, such a demonstration has not been made for the several kinds of curves in use for estimating hydrologic frequencies.

The literature of the application of probability methods to engineering data is practically confined to demonstrations of mathematical methods of curve fitting. The philosophy of the subject has been neglected, and the

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NOTE.—The paper by S. D. Bleich, Esq., was published in February, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: May, 1934, by Messrs. H. Alden Foster, J. J. Slade, Jr., Charles W. Sherman, Merrill M. Bernard, Clifford Seaver, and Jose Garcia Montes, Jr.

[21] Civ. Engr., Los Angeles, Calif.

[21a] Received by the Secretary July 20, 1934.

mathematical nicety of the familiar methods of treating skew frequency curves has too often been mistaken for a soundness of reason for their application to the problems at hand.

That the Pearson curves are not very satisfactory for use in representing hydrologic probabilities is indicated by several systematic departures of the curves from the data. When the same order of departure recurs in a large majority of cases investigation into the underlying causes is indicated as desirable and use of the curves should be made only with full recognition of their shortcomings.

Probably the most important of these systematic departures, and the only one the writer will attempt to discuss, is that of the extremely high values. A considerable number of long-term rainfall and run-off records contain more extremely high values than are indicated as probable by Pearson curves fitted to the data. An example of this is seen in Fig. 5. The 10-min duration curve indicates that an intensity of 9 in. per hr should occur, theoretically, 0.45 times in 100 yr, or once in 220 yr, whereas the actual record was once in 62 yr. The 15-min duration curve shows an intensity of 8 in. per hr to have a theoretical frequency of about 0.55 times in 100 yr, or once in 180 yr. Actually, this intensity was reached twice in the 62-yr period, or once in 32 yr, almost six times the theoretical frequency.

This same kind of departure may be noted in many widely scattered records, strongly indicating that major storms are controlled by a law of their own. If this is the case, the data of hydrology are heterogeneous and do not lend themselves to treatment by Pearson's constructions which are limited by him to homogeneous material.[22]

Even if heterogeneous it might be possible to group such data into sets of essentially homogeneous material, each of which might be treated by probability methods. If entirely heterogeneous some such treatment as that favored by the author would seem to be the most satisfactory. To the extent that his method gives frequencies for the higher intensities more nearly approaching those of the record, it is certainly to be preferred over present probability methods in studies in which the extreme values are sought to be determined within safe limits.

The writer clings to the idea that data of this kind can be handled properly by probability methods, but evidence continues to accumulate to cast doubt on the accuracy of present methods. The author's paper is an addition to this evidence. Surely it would be carrying faith in a method to an extreme to accept a theory that the probable frequency of occurrence of an event is only once in 180 yr when the event has actually occurred twice in 62 yr.

The author deserves the thanks of the profession for an interesting and understandable presentation of a study which contains much valuable information.

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[22] "Mathematical Contributions to the Theory of Evolution," by Karl Pearson. Draper's Company Research Memoirs, Biometric Series II, XIV, p. 4.

S. D. Bleich,[23] Esq. (by letter)[23a].—The remarks by Mr. Sherman bring out the fact that considerable work could be done to advantage on rainfall studies. While hydrologists appreciate the importance of such work, it is impossible to convince municipal authorities of its desirability. The criticisms made by Mr. Sherman are discussed in the order presented by him:

(1) The published records of the U. S. Weather Bureau Station in Central Park, New York City, were used. The records of the U. S. Weather Bureau Station at Battery Place, New York City, were studied independently, but this work has not yet (1934) been completed. The separate records of the two stations were not compared with each other. Further rainfall studies have been suspended indefinitely on account of lack of help, but it is hoped that economic conditions will improve sufficiently to allow a resumption of them.

(2) The Weather Bureau publishes the maximum rates of precipitation for 5, 10, 15, 30, 60, and 120 min, and for 24 hr. In the design of sewers for which purpose the formulas were primarily computed, the first six periods mentioned are of importance. Periods longer than 2 hr would be useful for water-sheds considerably greater than those contributing to sewers in New York City. For such areas, other formulas have been devised.[24] It is believed that the formulas given are sufficient for the use of all periods between 5 and 120 min, inclusive, and it would probably be safe to extend them to periods slightly longer than 2 hr.

(3) As stated, the printed records of the Weather Bureau were used and compared with the printed records of the Committee on Rainfall and Run-Off of the Municipal Engineers of the City of New York and with those collected by Mr. Hufeland, and only where discrepancies appeared were the original records consulted.

(4) All the formulas given in the paper are empirical and were designed to fit the given data as closely as possible with the form of equations used. Mathematical niceties and preconceived ideas of the values of the constants and exponents were avoided. The formula devised by Mr. Sherman, as well as the writer's unified Equation (8), may satisfy the æsthetic feeling, but would be difficult to justify logically. It is evident that, by not restricting the values of the constants, formulas would be obtained which will give results that correspond most nearly to the data, as is amply shown by the paper. It is for this reason that Equation (8) was not considered as good as the group of formulas given under Equation (5).

Rain storms are extremely complicated phenomena and are not of a single type. The conditions which are the principal underlying causes of rainfall intensities that occur as often as once a year probably differ from those causing the very severe intensities that occur only rarely, say, once in 15 yr, or in 50 yr. It is not to be expected that the same formula, or the same constants, will express all types of storms equally well.

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[23] Asst. Div. Engr., Board of Transportation, New York, N. Y.

[23a] Received by the Secretary July 6, 1934.

[24] "Formulas for Rainfall Intensities of Long Duration", by Merrill M. Bernard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 592; as well as "Frequency and Intensity of Excessive Rainfalls at Boston, Massachusetts", by Charles W. Sherman, M. Am. Soc. C. E., *Loc. cit.*, Vol. 95 (1931), p. 951.

(5) The rainfall records published in the *Journal* of the Municipal Engineers of the City of New York, as well as Mr. Hufeland's paper, were consulted, and were compared with the records of the U. S. Weather Bureau and only where there were discrepancies were the original records consulted.

(6) The formulas of the late Kenneth Allen, M. Am. Soc. C. E., were given careful consideration. They were designed in a different manner, however, from the writer's formulas; consequently, the results must be interpreted differently. For each duration Mr. Allen selected the heaviest rainfall that occurred in each year and neglected all the other intensities, even though some of them might be heavier than the severest rainfall intensity that occurred in other years. The results given by Mr. Allen's formulas[25] are described by him as giving "the probability percentage of years in which the given intensity of rainfall continuing over a given number of minutes will not be exceeded", whereas the writer's formulas, grouped together, give the frequency or the number of times in a given period of years that a rainfall intensity for a given duration, in minutes, has been equalled or exceeded.

Mr. Montes states that the storms embraced in the study are of a fixed character, some being due to one set of causes and others to entirely different causes. The writer admits his inability to segregate the storms in their proper classification so as to be able to treat each homogeneous set separately. Each kind of storm would probably have its own distinctive law and formula. The writer's problem has been a simpler one, namely, to obtain, as accurately as possible, the aggregate frequency of rain intensities for different durations for all types of storms.

Mr. Montes' discussion is very instructive and brings out important facts which are likely to be forgotten while the investigator is absorbed with the arduous calculations. The study deserves the time and efforts of a few qualified hydrologists, and large cities would find the expense for conducting this research comparatively small and the results very profitable in planning municipal improvements.

Use of the duration type of curve was attempted, as illustrated in the paper, but the result was not altogether satisfactory. Mathematical formulas corresponding to such curves are quite unwieldy.

Mr. Slade appears to be of the opinion that the probability method is the proper one for handling rainfall data. The difficulty with the method is that it cannot be justified rationally, particularly when a different coefficient of skew is used for each of the durations.

While it is ingenious, the probability method seems to be artificial and has only the appearance of mathematical logic. To use the values obtained by the skew probability method rather than the actual data in computing the modified exponential formula would result in values that would depart materially from the recorded values which no amount of mathematics could justify. If the physical laws were known, only then would there be good reason for using whatever calculations are necessary with the recorded data in order to obtain the numerical values for the constants in the formula which expresses the physical laws.

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[25] *Engineering News-Record*, April 7, 1921, p. 588.

In Equation (11) Mr. Bernard presents a uniform expression embracing rainfall intensity, duration, and frequency, which probably could be made to fit the recorded data better than the writer's unified Equation (8), as the former contains five parameters which are to be determined, and the latter only four. For this very reason and also because of the expression proposed for the formula, it would be very laborious to obtain the values of the five parameters which would give results corresponding closely to the given data.

Mr. Seaver has taken considerable pains to check Table 2 giving the chronological records of heavy rainfall intensities at the Central Park gauge, in New York City. He has also made instructive studies which clearly indicate that the results given by rainfall formulas will vary with the number of years of observations, thereby giving rise to the conjecture that the present results may not be final, but will have to be modified with the accumulation of additional data in the future. This is probably true, but, in view of the long period already covered by the Central Park gauge, it is not probable that the results will be varied materially by subsequent studies.

By means of Table 12, Mr. Seaver demonstrates clearly that two locations, only a few miles apart, may have materially different characteristics of rainfall intensities. While only a few years are covered by this comparison, it is probable that extended studies will confirm the truth of this finding. It is important that definite knowledge of this variation be obtained, which can only be accomplished by locating a large number of rain gauges at a number of points in each of the Boroughs of Greater New York City, to be accompanied by an exhaustive study of all the records, in order to learn the specific rainfall intensities and frequencies for each location. This would enable sewers to be designed so as to serve properly the areas where they are built, resulting in adequate drainage facilities without waste and in economy of construction which would more than offset the cost of such extended investigation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ANALYSIS OF SHEET-PILE BULKHEADS

Discussion

By D. P. Krynine, M. Am. Soc. C. E.

D. P. Krynine,[25] M. Am. Soc. C. E. (by letter)[25a].—Few engineers have analyzed the behavior of sheet-piling mathematically. Credit should be given to Dr. H. Blum who practically was the first to make such a comprehensive analysis; next was Dr. E. Lohmeyer.[2] A short history of the work of both Dr. Blum and Dr. Lohmeyer has been given by the writer elsewhere.[26] The paper by Mr. Baumann is a valuable contribution in this field.

For some reason the ingenious experiment represented in Fig. 2 was never duplicated and checked and, consequently, Mr. Baumann was obliged to base his analysis on a considerable number of additional assumptions. As far as some of the latter are concerned, the writer's opinion differs markedly from that of Mr. Baumann.

Conception of Passive Resistance.—If a rigid loaded plate is placed on the upper horizontal surface of a perfectly elastic body, it sinks as a result of elastic deformations, which disappear when the load is removed. In the case of excessive load, plastic deformations may take place, and the body may fail. The cycle of deformations in this case is: (1) Elastic; and (2) plastic. If an actual soil mass is loaded in the aforementioned manner, the settlement begins at once, even under a relatively light load; and it disappears only partly when the load is released. This does not mean that one part of the deformation is plastic, and another elastic; the explanation is that the non-reversible part of the settlement under a relatively light load is due mostly to a simple downward motion of particles which try to penetrate into the interstices between others. The cycle of deformations in this case is: (1) Simple motion of particles, which may be accompanied by partial elastic action; (2) elastic deformations; and, finally, (3) plastic deformations. The supporting power of the soil is due mostly to its shearing resistance, and not so much to its capacity to resist compression stresses. Actually, if there

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Note.—The paper by Paul Baumann, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by Jacob Feld, M. Am. Soc. C. E.; and August, 1934, by Messrs. R. L. Vaughn, M. A. Drucker, and Raymond P. Pennoyer.

[25] Research Assoc. in Soil Mechanics, School of Eng., Yale Univ., New Haven, Conn.

[25a] Received by the Secretary August 16, 1934.

[2] "Der Grundbau", von Brennecke-Lohmeyer, Fourth Edition, Vol. II, p. 77 *et seq.*

[26] "Comments on Failures in Sheet-Piling", by D. P. Krynine, M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 10 (October, 1933), p. 582.

were no shearing resistance at all, the soil would not be capable of supporting any load. This may be easily understood from the following analogy: Water is practically incompressible, but its bearing power is zero because it lacks shearing resistance entirely.

If a vertical wall is pushed into a sand mass, the succession of deformations is analogous to that of the preceding case: First, a simple motion of particles; next, elastic deformations; and, finally, plastic deformations. The passive resistance of the sand is its capacity to stand shearing stresses close to the end of Stage (2). Obviously, the stressed condition of the "passive prism" is more complicated than simple shear and consists also in some compression and other phenomena which are rather difficult to specify. A noteworthy feature is the stretching of a narrow zone along the shearing surface, which may be observed in the laboratory if one wall of the experimental box is made of glass. In the case of wet experimental sand, change of the moisture content in that zone may be easily observed by the naked eye. In the initial stage, (1), the passive resistance practically does not act. In the case of excessive horizontal load, the sand mass may fail along a shearing surface as indicated in Fig. 20.

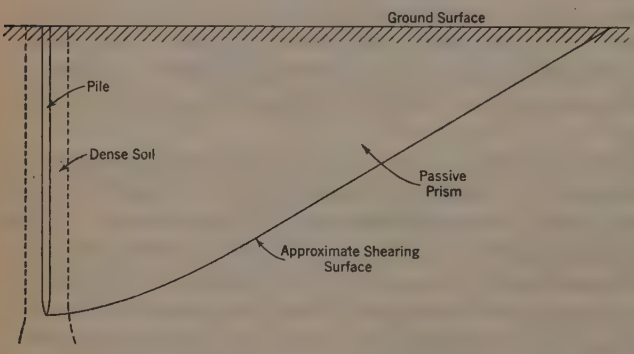

FIG. 20.

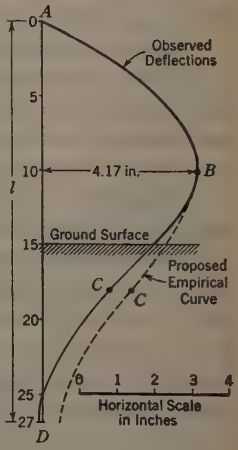

FIG. 21.

In the Blum-Lohmeyer method (Fig. 5) the passive resistance is assumed to act from the very beginning of the experiment, which the writer has shown in the preceding remarks to be untrue. In experiments such as those of Professor Franzius,[7] the passive resistance begins to act earlier than in natural sands, but at any rate later than is assumed in the Blum-Lohmeyer method. Professor Franzius' experiments were made on tamped sand, and his experimental data should be applied with certain care.

Compression of the Soil Close to Sheet-Piling.—Under the heading "Method of Graphical Analysis", Mr. Baumann expresses his belief that the ground adjacent to a bulkhead is compressed when sheet-piles are driven. This is perfectly true, but this circumstance cannot balance the initial "lost motion" of particles described previously. Actually, although the displacements of the adjacent soil which occurs in pile-driving, is equal to the thickness of the sheet-piles, the compressed zone is rather restricted and forms a kind of dense "envelope" around a pile or a sheet-pile (see Fig. 20). Several years ago the following experiment was made in the writer's laboratory. A pile model, 20 in. long, and somewhat more than 3 in. in diameter, was driven under field conditions into clay; then a cross-section through the center of the pile, 10 in. below its tip, was made, using sharp shovels and knives. One-half the earth mass was removed and another one-half extracted in the form of a monolith. The surface of that monolith was carefully studied optically by Dr. G. I. Pokrowski, using the reflectometer.[27] The light-dispersing capacity of the soil depends on its density; and it was found that at a distance of about two or three diameters of the pile the light-dispersing capacity and, hence, the density of the soil, were more or less constant and increased sensibly in the proximity of the pile. Thus, it was concluded that the width of the "envelope" in question, is rather insignificant. As far as the present case is concerned, this should mean that the process of sheet-pile bending was resisted by the dense soil very slightly, if at all. Fig. 20 shows diagrammatically the relative insignificance of that "envelope", as compared with the passive prism.

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[7] *Der Bauingenieur*, 9 Jahrgang, 1928, Hefte, 43 und 44.

Observed Deflections; Equation of the Elastic Line.—According to the preceding considerations, a part of the bending occurred when there was no passive resistance; and another part when that resistance was already acting. Perhaps this explains why the observed deflection ordinates were twice those of the computed ordinates (Table 2).

Equations (9) to (15) correspond to the fitting of empirical curves to the experimental deflections. Mr. Baumann states (under the heading, "Analytical Determination of Interlock Efficiency",) that, "the observed elastic line is closely approximated by two hyperbolic spirals with a common point and tangent at the point of minimum radius of curvature; that is, maximum moment." Furthermore, he states that the upper spiral (Distance *AB*, Fig. 21), becomes faulty near the top support, and that the lower spiral does not satisfy the boundary condition, namely, that at Point *C*, where z is about 18 ft., the radius of curvature should be infinite. Thus, both spirals are faulty, but Mr. Baumann believes that his empirical curve "closely checks the elastic line in the vicinity of minimum radius of curvature". Certainly this is true; but the shape of that part of the experimental curve is such that many other curves may fit it. Particularly, was the choice of spirals incorrect since a hyperbolic spiral winds steadily about its pole and never flattens so as to have an inflection point, such as Point *C*, Fig. 21. An empirical curve should fit the entire elastic line, and not only a part of it. For instance, the following curve, although far from being ideal, may serve that purpose better:

$$\Delta = k\left[2\tanh\frac{\pi z}{l} - \tanh\frac{\pi}{l}(z+\beta) - \tanh\frac{\pi}{l}(z-\beta)\right]\ldots(96)$$

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[27] "Soil Investigations in 1928-1929", by G. I. Pokrowski and N. V. Laletine (in Russian), p. 65.

in which, Δ = deflection, in inches; z = vertical distance, in feet, from the upper support, A (origin of co-ordinates); l = total length of the sheet-pile, equal to $28 - 1 = 27$ ft; and k and β = numerical parameters.

Should the curve of Equation (96) possess a maximum of $\Delta = 4.17$ in. at the point, $z = 0.375\ l = 10.1$ ft, the values of the parameters would be $k = 3.29$ and $\beta = 16.3$. In Fig. 21, the observed deflection curve is shown by a heavy solid line, and its probable extension by a light solid line. The empirical curve practically coincides with the observed deflection line between the points, A and B, and its lower part is shown by a dotted line. The value of Δ is not zero at $z = l$; but the curve would go exactly through the toe of the bulkhead if the z-axis were turned slightly. The curve of Equation (96) is used in electrical engineering for determining dynamic action of one coil of an electrical machine on another.[28]

Interpretation of the Permanent Deformations of the Test Piles.—Fig. 2 indicates that there were seven test piles facing the test wall. The deflections were measured on three outside sheet-piles, as shown in Fig. 2(*b*). Nothing is known about the differential deflection; hence, it should be assumed that the deflections on all the piles were uniform within some reasonable limits, and that the curve of observed deflections as shown in Table 2, represents an average.

The passive resistance, however, was not the same for each of those piles; it was greater for the center pile and less for the outside piles. This follows from the definition of passive resistance, which is shearing resistance of the soil along a certain surface. Such a shearing resistance, corresponding to the center of the tank, lay in a plane normal to the test wall (a two-dimensional problem); while, for the outside piles, the shearing surfaces could extend freely not only in front of the tank, but sidewise as well. The greater resisting force prevented the outside piles from excessive plastic bending in vertical planes, as is clearly seen in Fig. 6. This phenomenon agrees fully with a statement which was already advanced elsewhere[29], namely, an isolated pile is stronger against overturning than the same pile placed in a row with others.

However, Fig. 6 also shows that all the piles underwent plastic deformations; hence, the applicability of elastic theories, by Mr. Baumann's method, is not fully justified. Plastic condition may even have started at the very beginning of the experiment owing to hammer blows, when the piles were driven to refusal; it was probably only emphasized during the experiment.

If there were no interlocking action, the center pile would deflect during the experiment more than the others, but because of the rigidity of the entire system in horizontal planes, it could not do so. A striking analogy as to the influence of rigidity may be found in the settlement of structures. On certain soils a non-rigid structure (as, for example, an earth embankment) settles more in the middle than on the edges, while a rigid foundation slab settles evenly. Reverting to the given case, it should be stated that the elastic condition apparently was never destroyed in the interlocking joints; therefore, upon removal of the loads, the interlocking joints returned to the original shape, so far as the plastic deformations permitted. In other words, the joints acted as heavy elastic springs connected by weaker central parts of the piles.

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[28] "Electromagnetic Problems in Electrical Engineering", by B. Hague, 1929, p. 204 *et seq.*

[29] "Resistance of Sheet-Piling to Overturning", by D. P. Krynine, M. Am. Soc. C. E., and R. W. Abbett, Assoc. M. Am. Soc. C. E., *Civil Engineering*, Vol. 3, No. 5 (May, 1933), p. 263 *et seq.*

Consider a wall of, say, ten panels, each consisting of seven piles, exactly as shown in Fig. 2. Any panel of this wall (including those on the outside) would be weaker than the experimental wall in question, since the "three-dimensional" passive resistance would act only at the ends of that wall. Therefore, in practice, the data obtained in the experiment of Mr. Baumann must be applied with care.

Stress Distribution in Passive Prism.—The problem of passive resistance may be reduced to that of a vertical loaded plate acting on a soil mass bounded from above by a horizontal plane. This is a somewhat complicated particular case of the general problem, when a loaded plate acts on the horizontal surface of an infinite soil mass. It is necessary to solve the general problem for both non-plastic and plastic soils, after which it will be easier to deal with particular cases; and that general problem has not yet been definitely solved in soil mechanics.

The Proposed Method of Design. — As Mr. Baumann rightly states, the method of designing bulkheads proposed by Blum-Lohmeyer is defective in assuming the passive resistance to be static. The writer believes that the Blum-Lohmeyer method is undoubtedly a commendable step forward in comparison with old, quite empirical methods. It has little, if any, connection with the study of physical properties of soils, however, and, therefore, is rather unacceptable from the point of view of steadily developing soil mechanics. The method in question should be considered as a temporary one, which sooner or later will be abandoned; therefore, all complicated constructions (as, for instance, those shown in Fig. 7), should not impress the designer unduly. Mr. Baumann proves beyond question that passive resistance depends on both the depth of a given point and the movement of the wall. In Fig. 14 a fine geometric construction is given. It is to be regretted, however, that the author did not use modern soil mechanics to its fullest extent. For instance, his opinion as to plastic soils remains completely unknown; and such soils are widely dealt with in bulkhead construction.

Conclusion.—This discussion is not to be considered as a criticism of Mr. Baumann's work. On the contrary, he should be highly commended for his effort, patience, and accuracy in observations and computations. His work is a valuable contribution to the study of bulkheads, more fruitful than any other existing; and if incontestable conclusions cannot be reached as yet, this is easily explained by the extreme difficulty of dealing with the subject.

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DISCUSSIONS

EXPERIMENTS WITH CONCRETE IN TORSION

Discussion

BY MESSRS. FRANK M. RUSSELL, AND LESLIE TURNER

FRANK M. RUSSELL,[6] JUN. AM. SOC. C. E. (by letter)[6a].—To the writer's knowledge, this paper is the first of its kind to be published on research investigations performed in the United States. It is very timely as the subject of torsion is becoming of more concern than in times past, because of the thoroughness and exactness to which modern designing is being extended.

When concrete is stressed to failure in torsion the outer fibers are stressed much more than the inner ones, due to their distance from the axis of the cylinder. The stress in this fiber, then, is not the true intensity of stress in the entire specimen. Upton has derived a mathematical analysis by which the true intensity of shearing stress above the elastic limit may be obtained.[7] In the particular case of a round shaft his results reduce to a constant correction factor of 0.75; that is, if a specimen is stressed beyond its elastic limit Equation (11) should read:

$$v_m = 0.75 \frac{2\,M}{\pi\,\rho^3_e} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

Besides the previous investigations mentioned by the author, a more recent one, and one which covers the subject very thoroughly, was performed by Takenosuke Miyamoto, of Tokyo, Japan, in 1927.[8] Miyamoto even extended his investigations so far as to derive design formulas for reinforced concrete screw-piles. His work really continues and completes the experiments[8] which were started by Professor E. Mörsch, of Stuttgart, Germany. Miyamoto's tests include plain concrete cylinders, and cylinders reinforced, as follows: Spirals at 30°, 45°, and 60°; spirals at 45°, with axial rods; spirals at 60°, with axial rods; spirals at 30°, with rings; axial rods and rings; both inner and outer spirals at 45°; both outer and inner spirals at

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NOTE.—The paper by Paul Andersen, Assoc. M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1934, by E. Mirabelli, Assoc. M. Am. Soc. C. E.

[6] Jun. Engr., U. S. Bureau of Reclamation, Boulder, Colo.

[6a] Received by the Secretary August 9, 1934.

[7] "Johnson's Materials of Construction", Rewritten by Messrs. Withey and Aston, and Edited by F. E. Turneaure, Hon. M. Am. Soc. C. E., Seventh Edition, p. 21.

[8] "Torsion Tests with Plain and Reinforced Concrete Bodies."

[8] "Der Eisenbetonbau", von E. Mörsch, Sixth Edition, Vol. 1, Pt. II, 1929, pp. 323–335.

45° with axial rods; and inner spirals supported by axial rods. He also gives a mathematical interpretation of all his results. He agrees with the author in that the 45° spiral is the strongest, and, further, that the 30° spiral is the weakest of the three and that the 60° spiral has a strength between that of the 45° and the 30° spirals. Miyamoto also agrees with the author that rings; both inner and outer spirals at 45°; both outer and inner spirals at which he found to give the greatest resistance of all, however, were those reinforced with inner and outer spirals and axial rods.

The paper is quite complete, but more could be stated about concrete in torsion without reinforcement, in order to develop more knowledge concerning the behavior of plain concrete itself, as in the case of twist in a dam. Not from the standpoint of detracting from the subject of reinforcement, but from that of making the subject more complete, the writer wishes to state the results of his own research program on torsion of plain concrete.[9] The mixes used in this series of tests ranged in compressive strengths from 2 390 to 11 080 lb per sq in. In torsion these corresponding mixes tested 352 to 903 lb per sq in. Revised by Upton' correction factor for this case, the foregoing would mean that the true shearing stress at failure for these mixes would be 264 and 677 lb per sq in., respectively. A total of 100 cylinders, 12 in. long and 3 in. in diameter, were tested, using eight different mixes.

To obtain such a high strength concrete, the mixes were designed by the method first introduced by Inge Lyse,[10] M. Am. Soc. C. E. This method is based on the principle that the strength of concrete increases directly in proportion to the increase in cement content, provided the water and absolute volume remain constant. Four different mixes were adopted using cement-water (not water-cement) ratios of 2, 2.5, 3, and 3.5. Two sandstones were used as coarse aggregates, and for each of the four mixes of one there was a corresponding mix of the other. Both sandstones were of the Lyons formation, but one was red and the other white. They have different characteristics, however, the red sandstone having a specfic gravity of 2.50 and a percentage absorption of 1.5, and the white sandstone, a specific gravity of 2.56 and a percentage absorption of 1.0. Ideal Portland cement and a good local washed sand were used in the mortar. Table 4 shows the results of tests of a large percentage of the specimens all of which were cured under water and tested wet.

The writer's experiments agree perfectly with the author's conclusion which states that the modulus of elasticity in shear depends upon its ultimate strength, and that high strength concrete has a higher shearing modulus than low strength concrete. For strengths of concrete nearly twice that cited by the author, the writer found that the relationship was still a straight line. The white sandstone mixes tested somewhat higher than those of the red sandstone, but both showed straight-line relationships between ultimate strength and modulus of elasticity in shear, and were exactly parallel.

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[9] "Torsional Elastic Properties of Higher Strength Portland Cement Concrete", by Frank M. Russell, Jun. Am. Soc. C. E.; submitted as a thesis to the Univ. of Colorado (1933) in partial fulfillment of the requirement for the degree of Master of Science.

[10] *Engineering New-Record,* Vol. 107, No. 19, November 5, 1931, pp. 723–724.

TABLE 4.—RESULTS OF TESTS*

Series†	Cement-water ratio, by weight	Water-cement ratio, by volume	Slump, in inches	AVERAGE COMPRESSIVE STRENGTH, IN POUNDS PER SQUARE INCH		ULTIMATE SHEAR STRENGTH, IN POUNDS PER SQUARE INCH		ULTIMATE SHEAR STRENGTH CORRECTED, IN POUNDS PER SQUARE INCH		MODULUS OF ELASTICITY, IN SHEAR, IN THOUSANDS OF POUNDS PER SQUARE INCH†	
				28 days	90 days	28 days	90 days	28 days	90 days	28 days	90 days
RD......	2.0	0.75	1.0	4 550	5 570	442	483	331	362	1 440	1 670
RE......	2.5	0.60	0.6	6 500	7 710	557	690	417	517	1 600	1 880
RF......	3.0	0.50	0.6	8 710	9 990	628	773	471	580	1 830	2 020
RG......	3.5	0.43	0.2	9 720	10 470	618	835	538	626	1 730	2 030
WD......	2.0	0.75	1.0	3 940	5 530	470	532	352	398	1 770	2 000
WE......	2.5	0.60	0.8	6 070	7 630	635	731	476	549	1 980	2 180
WF......	3.0	0.50	0.8	8 350	9 810	673	727	505	549	2 160	2 290
WG......	3.5	0.43	0.2	9 560	11 080	773	903	580	677	2 160	2 270

* All strengths and modulus of elasticity in shear are in pounds per square inch.
† *R* denotes coarse aggregate of red sandstone; *W* = white sandstone.

Several relationships were studied to see how closely torsional properties could be assumed to parallel compressive properties. The theory of Professor Lyse determines the relationship between cement-water ratio and ultimate strength in compression, but it does not mention whether the straight-line relationship holds true for torsion as well as compression. At earlier ages (7 days) the straight-line relationship holds. As the concrete ages the trends are the same, but a greater increase in strength occurs between cement-water ratios of 2 and 2.5 than for any cement ratio greater than 2.5.

On a construction project it is quite probable that a part of a structure may have intermittent loads applied before it is thoroughly cured. In such a case, it would be well to know just how this might affect a member subjected to torsional stresses. With nearly every set of the writer's tests a pair of specimens was tested to a load which was from 40 to 60% of the ultimate for that set. These pairs were tested at each 7, 28, and 90-day period, and restored to water after each test. In twelve of fourteen sets the modulus of elasticity in shear was higher due to partial loadings with the greatest difference at 28 days. There was no apparent effect on the ultimate strength due to partial loading.

Along this same line of investigation, tests were made to find the effect of running the load up twice before taking the third trial to failure. The partial loads varied from two-fifths to two-thirds of the ultimate strength. The results from this showed that the properties are not affected materially, even if the proportional elastic limit is somewhat exceeded.

Further investigation found a relationship to exist between the modulus of elasticity in compression and the modulus of elasticity in shear. While the curve shows a definite curvature for each sandstone mix, it is slightly different for each one. Fig. 11 shows the curves superimposed upon the same axes. The formula for the red sandstone curve is:

$$E' = 1.20 + 0.058E + 0.003E^2 + 0.002E^3 + 0.0007E^4 + 0.0001E^5 \quad ..(32)$$

in which, E' and E are in millions of pounds per square inch. For the white sandstone, the formula is much simpler, being,

$$E' = 1.18 - 0.04E + 0.056E^2 \ldots\ldots\ldots\ldots\ldots\ldots(33)$$

The points plotted are the averages for each set of specimens tested. There is a possibility of one curve approximating all points plotted. It is quite probable that for other concrete mixes a similar relationship could be found, using this same principle. This is quite plausible from the fact that there is a definite relationship between modulus of elasticity and ultimate strength in compression, and between modulus of elasticity in shear and ultimate strength in torsion. Then, with the same concrete, there should be a definite relationship between the two moduli. Such a curve would be a good basis for choosing a suitable factor of safety, as suggested in the author's "Conclusions".

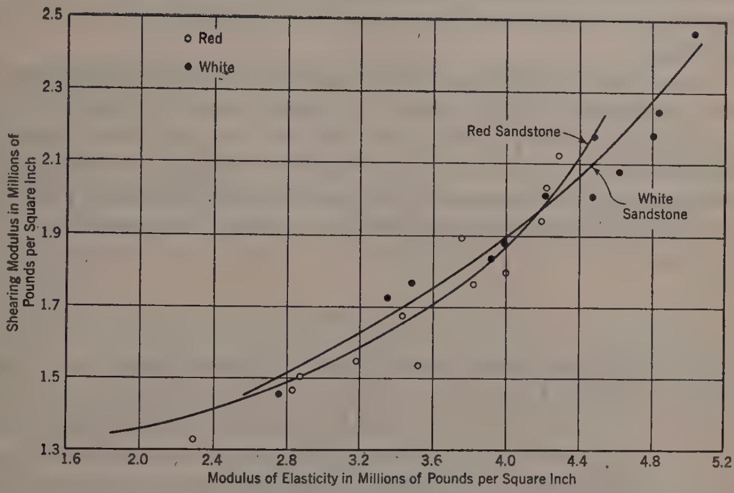

FIG. 11.—COMPOSITE OF ELASTIC CONSTANT CURVES.

LESLIE TURNER,[11] ESQ. (by letter)[11a].—A facet of concrete design as yet scarcely understood or appreciated, is opened for discussion in this paper. Doubtless, this topic has not been fully investigated owing to: (1) The complexity of the subject; and, (2) its lesser incidence and commercial importance vis-a-vis bending and direct stresses.

However, with more precision in design and with the more advanced types of structures, the effects of torsional resistance will be taken more and more into account, particularly when it becomes more widely appreciated that the influence of torsion on the factor of safety of reinforced concrete structures may be considerable. Consider, for example, the case of the simple slab spanning two directions on four walls; the torsional effects in thick slabs may be so large as to influence considerably the moment calculations. In all monolithic structures in which beams and slabs run into each other at right angles the torsional effect is always present, and is responsible to no mean degree for the additional security or factor of safety beyond the theoretical often found. At present, however, one cannot determine its amount with accuracy and precision, so that papers describing research on this subject are to be welcomed.

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[11] Cons. Engr.; Technical Adviser, Trussed Concrete Steel Co., Ltd., London, England.

[11a] Received by the Secretary August 9, 1934.

It is to be noted that the Tokyo experiments are not cited in the paper. These were carried out by M. Takenosuke about 1926 on ninety-six circular specimens 30 cm in diameter.[12] The results are in general accord with those of the author.

It is noted that Mr. Andersen has plotted the angle of twist against stress computed according to the conventional elastic theory; it would be interesting to have these angles plotted against torques. The reason for this is that the writer and Mr. V. C. Davies have found[13] the tensile stress-strain relation of plain concrete to be more nearly parabolic than straight, and may be expressed for the 1:2:4 concrete used, as: Stress, in pounds per square inch $= 244 - 450 \times 10^6 (0.000067 - \text{strain})^{1.5}$, or, in general terms,

$$f = t - C\,(D - d)^{1.5} \qquad (34)$$

in which, t = the maximum tensile stress; D, the maximum tensile strain; d, the strain at Stress f; and, C is a constant.

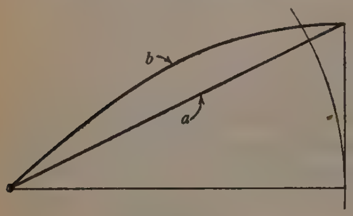

FIG. 12.

Thus, the stress distribution across a section follows Curve *b*, Fig. 12, rather than Curve *a*, the straight line. This is confirmed by Fig. 7, in which the tensile curve is parabolic in form. It differs from the writer's results, however, in the high value of the strains measured. The difference is more than would be accounted for by concrete mix, and it is difficult to explain. Contributory causes may be the shape of the section; the position at which the strain was measured; the influence of the reinforcement on the shrinkage of the concrete; and the fact that the tensile strain may be influenced by the presence of the diagonal compression in the torsion specimen when compared with a pure tension specimen. The values given in Table 2 for shear modulus agree quite well with the writer's results.

The foregoing disposes of the conventional elastic theory in relation to torsion and introduces what may be styled a hyper-elastic theory which modifies the formulas by which stresses are calculated for any particular section. The higher extensibility of concrete at high stresses, which follows from the parabolic tension curve, is a feature of considerable importance in torsion problems and explains another interesting discovery, namely, the lesser importance of the re-entrant angle in concrete sections, which, according to the conventional theory, is the vital spot. For most materials the stresses in the corner of a 90° angle, such as between the rib and tee of a beam, are infinite, whereas the writer's experiments have proved that this case does not hold for concrete although, of course, the addition of corner fillets does increase the strength of sections to certain degrees.

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[12] "Torsional Strength of Reinforced Concrete", by Miyamoto Takenosuke, *Concrete and Constructional Engineering*, Vol. 22, 1927.

[13] "Plain and Reinforced Concrete in Torsion", by L. Turner and V. C. Davies, *Selected Paper No. 165*, Inst. C. E., London, England, 1934.

The writer did not make or test any circular sections and he is glad to note Mr. Andersen's confirmation of his own conclusion, that the circular section is sensibly the same as the exscribed square in torsional strength. On account of its larger area, of course, the square is slightly stronger than the circle.

The circular section with spiral reinforcement is the only one that lends itself to mathematical investigation. The basis for calculation for other sections is so obscure at present as to render purely theoretical treatment almost valueless. Furthermore, the practical difficulty in providing generally any but rectilinear reinforcement (longitudinal bars and links or ties), should not be overlooked, while the value of such reinforcement is still more difficult to assess theoretically. It is far less effective, of course, than spiral reinforcement.

The writer has preferred, therefore, to base his recommendations on the results of tests, with empirical formulas and rules for application to other and intermediate sections, shapes, and reinforcement. Rectangles, T-beams and L-beams have been considered, and, in this connection, it should be mentioned that the soap film analogy developed by Messrs. A. A. Griffith and G. I. Taylor,[14] has been very useful in pointing the way toward the solution of these problems. The question of torsion combined with bending and direct forces, of course, requires much further thought and research.

Mr. Andersen has developed some interesting formulas on the assumption that peripheral reinforcement is provided to take care of the tensile stresses in excess of the permissible stress for plain concrete. The question, however, is what is the permissible stress and what relationship should it bear to the ultimate to give the necessary factor of safety. In discussing Fig. 10 (see "Concrete in Torsion"), it is suggested that the concrete in a reinforced section can sustain a higher tensile stress than that in a plain section. While this may be "apparent", due, among other causes, to the reinforcement spreading incipient cracks, or to shrinkage causing stressing before torque is applied, it is difficult to accept different qualities for the same concrete.

The formulas could be adapted for the parabolic distribution, and then the factor of safety must be selected dependent upon the amount of external or visible cracking (if any) allowed. This would bring them into line with bending formulas where cracking is theoretically permitted in tension areas although, of course, it does not often develop, practically speaking. In other words, the cracks, if formed at all by the concrete yielding its stress to the steel, are so minute and dispersed as not to be noticeable.

The stress distribution along a side shown in Fig. 3 is the conventional form resulting from St. Venant's theory for elastic bodies, but it is more than doubtful that it applies to concrete except in the very early stages of stressing.

Finally, in welcoming heartily a paper on this somewhat involved subject, the author is to be thanked and congratulated on introducing new material and methods of attack and on presenting them so concisely.

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[14] "The Use of Soap Films in Solving Torsion Problems", *Proceedings*, Inst. Mech. Engrs., London, England, 1917, p. 787.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRESSES IN SPACE STRUCTURES

Discussion

By A. H. Finlay, Assoc. M. Am. Soc. C. E.

A. H. Finlay,[8] Assoc. M. Am. Soc. C. E. (by letter)[8a].—The concise presentation of Professor Mayor's ingenious method of analysis of spatial stresses contained in Professor Constant's paper is extremely interesting and instructive—the more so because the author avoids the language of projective geometry. The idea of evaluating concurrent, non-co-planar forces by first evaluating a closely related set of non-concurrent, co-planar forces is novel and greatly facilitates analysis. The conventional method of analyzing space structures has too often been demonstrated using such simplified loadings that many of the real difficulties of a complete analysis of such structures have been concealed. As the author states, the advantages of the proposed method increase with the complexity of the structure and loading.

As a simplification, the central axes for all systems at any joint could be taken through the joint instead of at different arbitrary distances, d, from it, as in Fig. 1. If this is done all the conjugates at a joint will lie in the horizontal plane containing the joint instead of in horizontal planes at varying distances above or below it. There would then be no need to complicate the picture by introducing the couple, S, which, of course, is merely the vectorial sum of the couples, S_0 and $R(d)$; nor would there be any necessity to refer to the projections of the conjugates upon a common horizontal plane, as the author felt impelled to in the paragraph containing Equation (5). Lastly, as will be shown, this simplification permits a shorter and more easily followed derivation of the expression for the distance, b.

The physical picture is simple: It is desired, for purposes of analysis, to combine each of the forces acting at a joint of the space structure with one other force so that in place of each of the original forces there will be a pair of forces equivalent to a system, composed, for convenience, of a vertical force, R, and a couple, S_0, acting in a horizontal plane containing the joint.

Note.—The paper by F. H. Constant, M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1934, by Messrs. William O. Osgood, L. E. Grinter, and Charles M. Spofford.

[8] Asst. Prof., Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

[8a] Received by the Secretary August 21, 1934.

Fig. 7 illustrates the foregoing: *ABCD* is a horizontal plane containing Joint *O*. Vector *F* is any force, bar stress, or load, acting at the joint. It is shown resolved into its vertical component, *R*, and its horizontal component, *F* cos ϕ. The additional force that must be combined with *F* so as to form a system composed of a vertical force, *R*, and a couple, S_o, in the plane, *ABCD*, must be *F′*, which must be equal and opposite to *F* cos ϕ and must act in the plane, *ABCD*, parallel to the horizontal component of *F* and distant *b* from it. The following relations exist, remembering that $S_o = R\,a$:

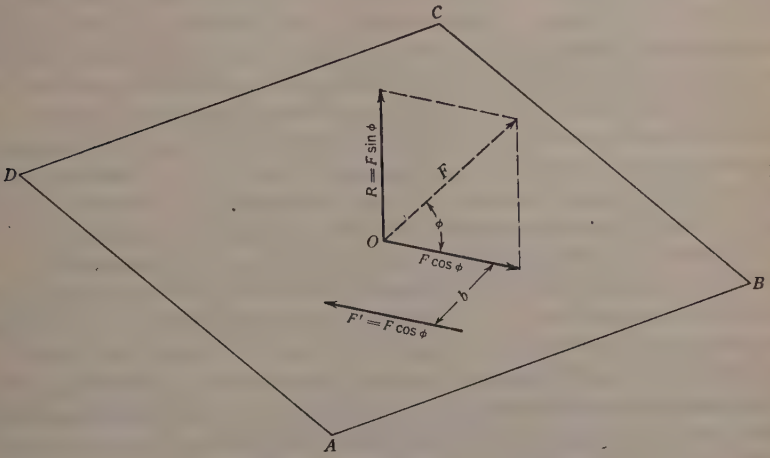

FIG. 7.

$$S_o = F'\,b = F \cos \phi\; b \qquad (14)$$

$$R\,a = F \sin \phi\; a \qquad (15)$$

and,

$$b = a \tan \phi \qquad (16)$$

Equation (16) fixes the line of action of the force, *F′*. The pair of forces, *F* and *F′*, is equivalent to the system, *R* and S_o. Each of these systems ("screws" in the language of statics) is tied to the other systems at the joint, as has been stated by the author, by the fixed relation, $\frac{S_o}{R} = a$. This quantity is often referred to as the pitch of the screw; its value is fixed arbitrarily in the present case at the dictates of convenience, but must be the same, of course, for all systems or screws at the same joint. The author has demonstrated clearly that these additional co-planar forces, or conjugates, *F′* (of which there are as many for any joint as there may be loads and bar stresses), are themselves in equilibrium and upon this condition, of course, the whole method rests.

The sign convention adopted by the author, and its resulting rule for aiding in the location of the lines of action of the conjugates, are both clear and easily applied.

As Professor Constant states, the conjugates of forces that lie in the same plane intersect in a point. Possibly this is obvious, but a brief proof may not be out of place: Choose any two forces meeting at any joint. Their conjugates will intersect somewhere. If any other force acting through the joint lies in the plane of the first two, it may be resolved into components coinciding with their lines of action and these components, therefore, will have conjugates coinciding with the original conjugates, thus demonstrating the statement. Had the two original forces lain in the same vertical plane their conjugates would not have intersected, but would have been parallel. The full statement would appear to be that the conjugates of forces lying in the same plane will either intersect in a point or be parallel.

The example chosen by the author illustrates the method clearly without introducing complexities due to unusual bar arrangement. The loading is complex, and yet the solution is simple and rapid. Whether the vertical reactions are more easily computed, using the moments of the diagonal and column conjugates as suggested in the paper, or whether it is more convenient to find them by summing the vertical components of the diagonal and column stresses, is probably a matter of individual preference.

The method furnishes a graceful solution of a problem which in the ordinary way presents annoying difficulties. It is in no sense a trick method although at first glance it may so appear. The conjugate forces, which are so simply related to the desired bar stresses, are merely co-planar forces in equilibrium and, as such, are subject to the three simple equations, $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma M = 0$, expressing this state. That these equations, susceptible as they are to graphical presentation, are more readily applied than $\Sigma X = 0$, $\Sigma Y = 0$, and $\Sigma Z = 0$, makes the method, in the writer's opinion, of real value, and Professor Constant is to be congratulated upon having made it it available in English.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

STREET THOROUGHFARES
A SYMPOSIUM

Discussion

By H. Shifrin, M. Am. Soc. C. E.

H. Shifrin,[12] M. Am. Soc. C. E. (by letter)[12a].—This Symposium, and especially the Report of the Committee of the City Planning Division on Street Thoroughfares Manual on Definitions, General Principles, and Recommendations, should prove valuable to engineers interested in city planning or planning for metropolitan areas. Most of the principles can be applied profitably to those new communities that are now (1934) being set up by the Federal Government, such as subsistence homesteads, and those being constructed to house the personnel connected with the immense power projects under way. This use, however, even if basically sound, can not be applied so readily when considering relief measures necessary in connection with old established metropolitan areas and large cities.

In the definitions furnished by the Committee, it would seem that the term, "Freeway" is synonymous to "Expressway". An expressway providing a highway for its express traffic exclusively, in order to be efficient, could not have access to it from abutting property. With no such access, the expressway really becomes a private right of way for through uninterrupted traffic, with practically no intersections at grade, and with no access to it from the abutting property. Such a definition would make the expressway synonymous to a freeway. One of these terms should be sufficient, as use of more than one for the same purpose merely adds to confusion.

The writer agrees thoroughly with the statements of the Committee (see "General Summary") that simply to widen existing streets to extraordinary widths does not provide the desired results. Experiences of engineers in cities in which extensive widenings have occurred during the 10-yr period from 1920 to 1930, have clearly proved that the cost involved in widening

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Note.—The Symposium on Street Thoroughfares was published in August, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[12] Cons. Engr., St. Louis, Mo.

[12a] Received by the Secretary September 13, 1934.

streets to extraordinary widths without providing for uninterrupted traffic at street intersections can not be justified on the basis of the benefits received by the terminals served through the widened streets, by the abutting property, or by the users of the thoroughfares. Experiences gained as a result of these widenings, have also proved that the methods used for determining benefit assessments to defray the expenses involved are often unjust and inequitable.

In most old cities of the United States, property values in the down-town or loop district, which is the main terminal toward which traffic is directed, are, in general, too high to permit the construction of satisfactory express or super-highways which would contain lanes for the uninterrupted movement of express and mass traffic through such terminal areas. As a matter of fact, the more feasible plan for serving these areas is to provide thoroughfares along the extremities or limits of such areas, and to utilize the existing facilities through the interior of the loop district. With sufficient thoroughfares of the expressway type connecting the valuable center or the down-town area on all sides with the highways or arterial streets existing in the metropolitan area, the stabilization of property values (which is the crying need of most cities) could be accomplished.

The policy of the highway departments in most States in the past has excluded the construction of highways or thoroughfares, or the use of highway funds for work within large cities, notwithstanding the fact that a substantial percentage of the funds made available is received from the citizens residing in such cities. That this fact is being realized is evidenced by the number of highway departments which are setting aside a certain portion of their funds to provide extensions of highway facilities, not only within the metropolitan areas, but within the corporate limits of the municipalities. This attitude has been further encouraged through the regulation of the U. S. Bureau of Public Roads in the expenditure of the Public Works funds set aside for highways, which regulations require that at least 25% of such funds be spent in cities. Use of such funds should properly be made only for the construction of expressways or freeways connecting the arterial or super-highways with the valuable centers or down-town districts of large cities, and no part of such funds should be used on ordinary city streets.

In addition to expressways or freeways, it would seem necessary that within the city proper, definite provisions should be made for mass transportation. Unless super-highways are made available, where a portion of the highway is physically segregated for the use of mass transportation only, it would seem desirable to designate certain existing thoroughfares of ordinary width for use of mass-transportation vehicles primarily. The satisfactory solution of providing mass-transportation facilities is urgent in practically all the cities of this country, and has been further aggravated by the recent widening of streets where no segregation has occurred to separate fast-moving traffic from mass transportation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

EFFECT OF SECONDARY STRESSES UPON ULTIMATE STRENGTH

BY JOHN I. PARCEL,[1] M. AM. SOC. C. E., AND
ELDRED B. MURER,[2] JUN. AM. SOC. C. E.

SYNOPSIS

An analysis of the action of members of a bridge truss subjected to axial stress and secondary bending that arises from the deflection of the truss in its own plane, is given in this paper. The study is devoted primarily to the question of the effect of secondary stress in reducing the ultimate strength of such members, and, therefore, particular attention is given to the redistribution that occurs when the outer fiber stresses approach the yield point. As a supplementary study, a series of laboratory tests was made to determine the actual behavior of compression members with thin walls subjected to high secondary stresses and loaded to failure.

From the general analysis, and from these tests, it is concluded that for types of members and loading conditions investigated (which are believed to simulate closely the essential conditions for most bridge members), the ultimate strength is practically unaffected, even by high secondary stresses, if, in the case of compression members, the relative wall thickness is maintained at the ratio ordinarily required by the leading standard specifications.

INTRODUCTION

The problem of secondary stresses has occupied a prominent place in the theory of structures since the original investigations of the subject by Winkler, Asimont, Engesser, and Manderla in the late Seventies. A correct and complete analysis was presented by Manderla in 1879.[3] Throughout the half

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NOTE.—Discussion on this paper will be closed in **February, 1935**, *Proceedings*.

[1] Prof. of Structural Eng., Univ. of Minnesota, Minneapolis, Minn.; Cons. Engr. (Sverdrup & Parcel), St. Louis, Mo.

[2] Detailer-Insp., Sverdrup & Parcel, St. Louis, Mo.; formerly, Research Fellow in Structural Eng., Univ. of Minnesota, Minneapolis, Minn.

[3] "Die Berechnung der Sekündarspannung welche im einfachen Fachwerke infolge starrer Knotenverbindungen auftreten", *Allgemeine Bauzeitung*, 1880.

century following, the problem has been the subject of many researches, both analytical and experimental, directed in the main toward clarifying and simplifying the analysis and to its experimental verification. To a considerable extent this effort has been successful. The theory of secondary stress analysis is now generally accepted by most authorities on the same footing as other phases of statically indeterminate stress analysis, and although the numerical calculations involved are tedious, various permissible simplifications have rendered the method quite workable for office design.

American engineers became generally interested in the secondary stress problem much later than European engineers, probably due in some degree to the predominance of the pin-connnected truss (for which, if all connections are true hinges, the secondary stresses are negligible) in American practice, and in part to a general distrust of the refinements of statically indeterminate stress analysis which at an earlier period was rather widespread. However, the last two decades (1914 to 1934) have witnessed a striking change in bridge engineering practice. The pin truss is no longer the dominant type; all spans from the smallest to the largest are being built as partly or fully riveted trusses, and the problem of secondary stresses has become a leading question in structural design. Most specifications now require such stresses to be computed for all sub-paneled trusses and for other cases in which there is reason to suspect that they may be large. That for many riveted bridge trusses of the massive type the secondary stresses are high is well established, theoretically and experimentally. In many cases the unit stresses due to secondary bending will reach 60 to 100% of the primary unit stresses, and in certain extreme cases they may exceed these limits.

The present method of providing for secondary stresses in design is subject to rather wide variations. Usually, a considerably increased unit stress (25 to 35% above the normal) is allowed for combined secondary and primary stresses and the member is proportioned by,

$$f = \frac{S}{A} + \frac{M\,c}{I} \qquad \text{.........................} (1)$$

in which, f = unit stress; S = primary tension or compression; A = cross-section area; M = bending moment; c = distance from the neutral axis to the extreme fiber; and I = moment of inertia. More commonly, perhaps, a blanket allowance, applied alike to all members, is provided in the prescribed unit stresses.

In a number of monumental riveted bridges (notably the Quebec (cantilever), the Sciotoville (continuous truss), and the Hell Gate (arch) Bridges), elaborate and more or less expensive special devices in fabrication and erection were used to reduce the secondary stresses to a point at which little or no excess material was required.

While it is thus clear that the possible occurrence of high secondary stresses has been widely recognized and the best standards of practice have required that these shall either be provided for in the design or largely eliminated in fabrication and erection, it does not appear that any considerable attempt has been made to evaluate the effect of secondary stress upon the actual ultimate strength of a member.

It is clear that the action of combined primary and secondary stress is a different phenomenon from that arising from direct stress and flexure due to applied loads. This will be elaborated later in the paper; it may be noted here that (*a*) secondary stresses are not required to maintain the equilibrium of a bridge truss; and (*b*) that they are induced by the relative joint displacements of the structure, which, in turn, are conditioned by the distortion of the truss as a whole; and when these displacements have occurred, there is no further tendency for the stresses to increase. This clearly is quite a different condition from that which obtains in, say, an eccentrically loaded column in which the deflection and moment, at high stresses, increase much faster than the load. This peculiarity of secondary stress action has been noted.[4] Indeed, the opinion has been advanced[5] that even a high percentage of secondary stress, resulting in extreme fiber stresses beyond the proportional limit, will have little effect in reducing the ultimate strength of the truss, since the increased fiber deformations occurring in the neighborhood of the yield point tend largely to relieve the secondary stress. While this extreme view has not been generally accepted by the profession, the subject is believed to merit more attention than it has thus far received.

It is a practically universal principle in structural design that all fiber stresses shall be kept well within the yield point of the material. This, however, is subject to certain exceptions; for example, bearing stresses on rivets and pins are generally permitted to run 50% in excess of the stresses on the main sections. If this specification is sound and if the structure is designed consistently, it means that such local stresses may pass the yield point without endangering the safety of the structure. It is also common practice to permit unit stresses in the stiffening trusses of suspension bridges greatly in excess of the stresses in the towers or in the cable, if comparable material is used in the latter. The logic of this practice is that the stiffening truss is not a main carrying member and that if loaded beyond the yield point, the bridge may still be in no danger of actual failure.

It is scarcely to be presumed that stresses beyond the yield point in either of these cases would ever be regarded as other than undesirable, but since such a condition would not produce structural collapse, it is felt that a smaller margin of safety is permissible than would be the case for the main carrying members.

A somewhat similar argument might well be advanced in regard to secondary stresses if it can be shown that, under the normal range of conditions, such stresses do not actually reduce the ultimate carrying power of the members. It may still be regarded as undesirable to have any considerable portion of the section stressed beyond the yield point, but it may be permissible to tolerate much higher limits for such stresses if this in no way endangers the safety of the structure. The remainder of the paper is devoted to a consideration, in some detail, of the relation of secondary stress to ultimate strength.

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[4] See, for example, "Theorie und Berechnungen der Eisernen Brücken", von F. Bleich, pp. 424–486; discussion by Edward Godfrey, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 89 (1926), p. 193; and Second Progress Report of Special Committee on Steel Column Research, *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1220 *et seq.* A clear statement of the limitations of secondary stresses will be found in "Modern Framed Structures", by Johnson. Bryan, and Turneaure, Pt. III, p. 14. No use is made of this, however, in the later discussion of the reduction in ultimate strength of columns due to secondary bending (see pp. 57–59).

[5] "Theorie und Berechnungen der Eisernen Brücken", von F. Bleich.

ANALYSIS OF PROBLEM

General.—Secondary stresses, as considered in this paper, arise from the displacement of the joints in the plane of the truss when the latter is subjected to external loads. If the members are connected by perfectly smooth hinges (and are non-continuous at all joints) no secondary stresses can develop. If, however, the members are connected by riveting (or welding) to gussets so as to form a practically rigid joint, there will be, in general, some degree of restraint at the ends of each member, and corresponding bending moments will be developed. The secondary end moments in any truss member, *m-n*, may be expressed by the well-known slope-deflection equation:[6]

$$M_{mn} = \frac{2\,EI}{L}\,(2\,\theta_m + \theta_n - 3\,R) \quad \ldots\ldots\ldots\ldots\ldots (2)$$

in which, θ represents the angular displacements of the joints referred to their original positions, and $R = \left(\frac{\Delta}{L}\right)$, the angular displacement of the line, *m-n*, each in radians. If τ_m and τ_n represent the angles between the end tangents and the line, *m-n*, at *m* and *n*, respectively, the equation for the secondary moment becomes,[7]

$$M_{mn} = \frac{2\,EI}{L}\,(2\,\tau_m + \tau_n) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

which is the form originally proposed by Manderla.

It may be well to call attention here to two basic characteristics of secondary stress action:

(*a*) Even when the secondary unit stress is a high percentage of the primary, the secondary moments offer no appreciable assistance in carrying the loads, and the members are always designed on the basis of full hinge action at the ends.

(*b*) The quantities, θ and R, in Equation (2) are computed from the linear displacements, Δ, of the truss joints, and for all cases of any practical importance they are directly dependent upon the axial distortions of the various truss members and, therefore, are linear functions of the loads.

The influence of secondary bending upon the ultimate strength of members depends importantly upon the type of stress acting on the member concerned. The several classes are considered separately in the following discussion.

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[6] "Abhandlungen aus dem Gebiete der Technische Mechanik", von O. Mohr, 1906, p. 422 *et seq;* see, also, "Statically Indeterminate Stresses", by John I. Parcel and George A. Maney, Members, Am. Soc. C. E., Chapter VII.

[7] See "Modern Framed Structures", by Johnson, Bryan, and Turneaure, Pt. II, pp. 427–428.

Tension Members.—In Fig. 1 is shown a tension member, m-n, acted upon at the ends by the moments, M_m and M_n. At any point, x, distant from the left end, the moment is (considering clockwise moments positive),

$$M_{(x)} = M_m - \frac{M_m + M_n}{L}\, x - S\, y \;\ldots\ldots\ldots\ldots\ldots (4)$$

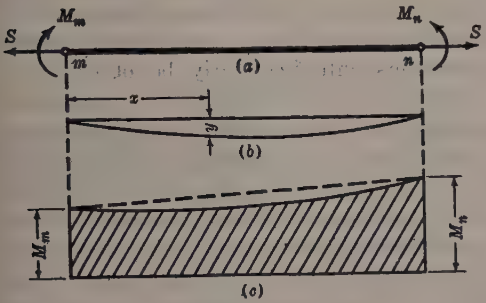

FIG. 1.

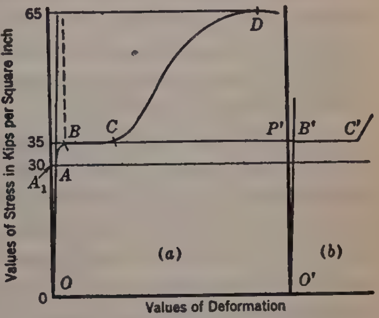

FIG. 2.

For the particular case in which the end moments are equal, and oppositely directed,

$$M = M_m - S\, y \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

It is clear that the maximum fiber stress occurs at the end of the member where the larger moment is applied, and (if this is the m-end) is equal to,

$$f = f_{\text{axial}} + f_{\text{bending}} = \frac{S}{A} + \frac{M_m\, c}{I} \;\ldots\ldots\ldots\ldots\ldots (6)$$

Since the relation, $f_b = \dfrac{Mc}{I}$, is derived on the assumption of linear stress variation, Equation (6) is not strictly correct beyond the proportional limit. When the yield point is reached and a large plastic flow takes place, the formula becomes quite inapplicable. The general tendency of the re-adjustment is toward an equalization of the tensile and compressive fiber stresses, respectively, such that a given maximum extreme fiber stress corresponds to a larger resisting moment.

Assume that Equation (3) is valid, then, as long as no stresses exceed the proportional limit, the behavior of the member is practically the same whether the end moments are due to eccentricity of axial loading ($M_m = S\, e_m$; $M_n = S\, e_n$), or to secondary bending. When the region of plastic flow is reached, however, the behavior is fundamentally different. In the first case, while the extreme fiber stress is relieved by the stress-strain re-adjustment, a total resisting moment equal to $S\, e$ must be developed regardless of the state of strain. In the latter case, since it is the tangential displacements that are invariably proportional to the loads (as long as $\dfrac{S}{A}$ remains within the proportional limit), and since a given angular displacement corresponds to a smaller bending moment when the stresses in the outer fibers approach the yield point, it is clear that within this region the secondary moments increase more slowly than the loads. Some elaboration of this point may be desirable.

Fig. 2(*a*) is a typical stress deformation curve for mild steel. Point *A* marks the limit of proportionality; Point *B*, the yield point; Segment *B*-*C*, the region of marked plastic flow; and Point *D*, the ultimate strength. Of course, Point *A* cannot be fixed accurately since there is no clearly defined limit of linear elasticity; slight deviations will frequently be observed at fairly low stress. However, it is not usual to find variations from proportionality of any considerable magnitude until the stress reaches the neighborhood of the yield point, as indicated in Fig. 2(*a*). For purposes of approximate calculation, a conventionalized stress-strain diagram, as shown in Fig. 2(*b*), is sometimes used.[8] This graph shows, roughly, that when the stress reaches the yield-point value at Point *B′*, no further increase in stress can occur until a deformation equivalent to the length, *B′ C′*, has taken place. This deformation may be as much as ten to fifteen times the length, *P′ B′*, the value corresponding to linear elasticity.

It is evident from the foregoing relations that in the case of a beam in flexure, if plane sections are assumed to remain plane, the outer fiber cannot be stressed beyond the yield point until the fiber stress, f'_b, over the greater part of the depth of the section has reached this stress limit. Referring to Fig. 3, a beam of symmetrical cross-section is assumed to be loaded so that the outer fiber stress, *QP*, is just up to the yield point value (Line *O′ P′* in Fig. 2(*b*)). If the load is further increased, the stress curve takes the form of Area *OPQT* in Fig. 3. It is particularly to be noted that the contribution to the beam deflection of a vertical slice of thickness, *dx*, at Section *POR*, in the two cases will be in the proportion of *PQ′* to *PQ*, while the moments will have the proportion of *PQTO* to *PQO*. It is clear from this illustration that when the outer fibers reach the yield point, a wide range of beam deflection is possible without any appreciable increase in the

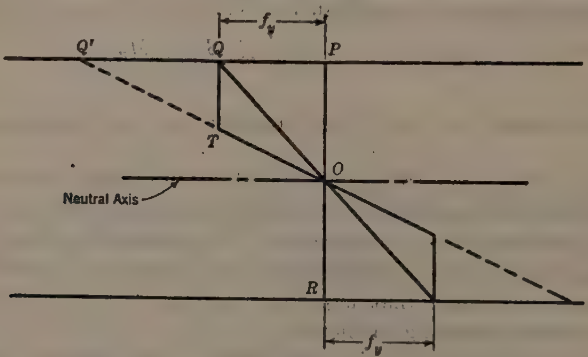

FIG. 3.

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[8] "Excentrische beanspruchte Säulen", von A. Ostenfeld, *Mitteilungen No. 3*, Lab. für Baustatik, Technische Hochschule, Copenhagen, p. 20.

extreme fiber stress. Since the angular rotations at the ends of a member can increase no more rapidly than the axial loads, it will be impossible even with high secondary stresses for the extreme fiber stress to exceed the yield-point value without the distortion increasing far beyond any value reasonably to be expected in a truss in service. A simple example may be taken to illustrate the point.

It will be assumed that a truss member, *m-n*, of symmetrical cross-section, is subjected at the end, *m*, to an average axial tensile stress $= f_p = \frac{S}{A}$, and a secondary stress $= f_b = \frac{M_m c}{I}$, in which, M_m is the larger of the secondary end moments. Then, the maximum stress in the member will be at the end, *m*, on the side of the tensile flexural stress and will be given by Equation (6).

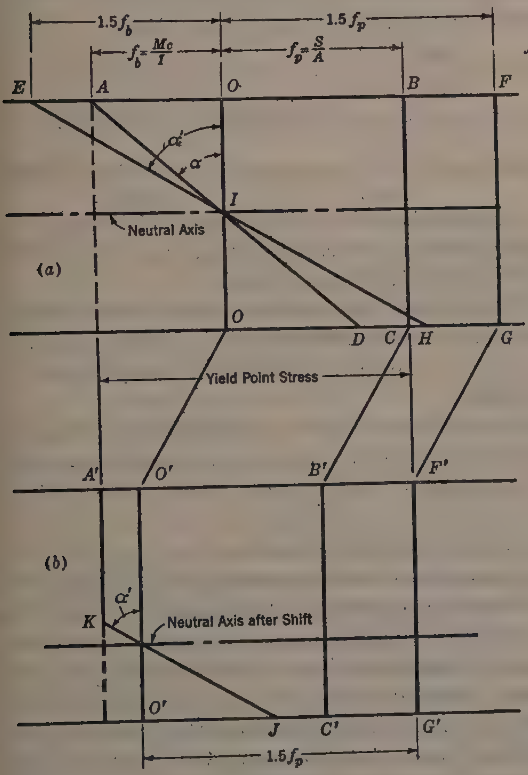

FIG. 4.

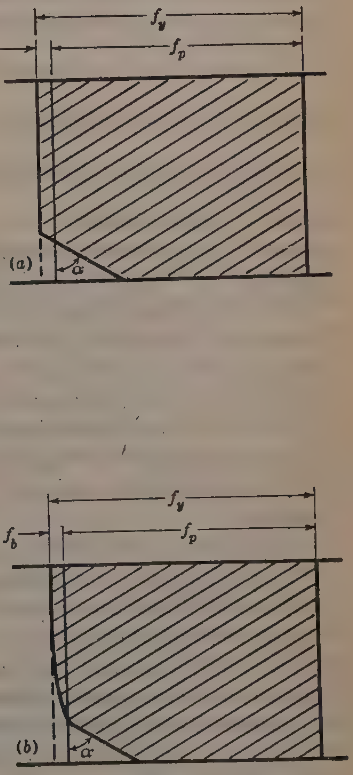

FIG. 5.

The stress graph is indicated by Curve *ABCD* in Fig. 4(*a*). Let it be assumed as a particular case that f_p = 20 000 lb per sq in.; $f_b = 0.75 f_p$ = 15 000 lb per sq in.; and that the yield-point stress, f_y, is 35 000 lb per sq in. The maximum fiber stress, *AB*, is then just at the limit beyond which plastic flow takes place.

Let it now be assumed that f_p receives an increment of 50%, bringing it to 30 000 lb per sq in. If the linear relation of stress to strain was to hold, f_b would become $1.5 \times 15\,000 = 22\,500$; the total maximum extreme fiber stress would be 52 500; and the stress graph would be the curve, *EFGH*, as shown in Fig. 4. On the other hand, assuming a yield-point stress of 35 000 lb per sq in., it is clear that this value cannot be exceeded until the extreme tensile fiber has suffered a stretch of approximately ten times that corresponding to 35 000 lb per sq in. It is obviously impossible, in a section of ordinary depth, for any such extreme deformation to take place unless the entire section is stressed to the yield point. Therefore, a redistribution of stress must be expected on the tensile side similar to that shown in Fig. 3. Again, it is noted that, as long as the average axial stresses are below the yield point, the tangential deflection angles (τ of Equation (3)) are strictly proportional to the primary unit stresses. In this case (for the small angles involved in the elastic deflections of steel structures), the angle, τ, should be increased in the proportion of $\frac{EO}{AO} = \frac{\alpha'}{\alpha}$. The actual stress graph as modified by the redistribution is the curve, $A' F' G' JK$, as indicated in Fig. 4(*b*). Even if a much higher ratio of $\frac{f_b}{f_p}$ is assumed, the modified stress graph is quite similar. As f_p approaches the yield point still closer, the secondary stress becomes negligibly small as indicated in Fig. 5(*a*), the neutral axis shifting toward the side of compressive flexural stress.

The preceding discussion is based upon the conventionalized stress-strain diagram of Fig. 2(*b*). If the more correct relationships of Fig. 2(*a*) are used, a stress graph similar to that shown in Fig. 5(*b*) results (for f_p slightly below the yield-point stress).

It may be concluded from this analysis that the redistribution of stress which occurs in the neighborhood of the yield point of a ductile material, such as structural steel, will practically nullify any effect of secondary bending.

Fig. 6 shows the elastic curve and moment diagram for a bar under a primary tension, *S*, and equal secondary end moments, *M*. Within the proportional limit the moment diagram appears as Area *abcd* in Fig. 6(*b*). As the extreme fiber stress (which is maximum at the end) passes the proportional limit, and the material begins to flow, a much smaller moment corresponds to a given rotation. Since it is the latter which increases in proportion to the primary stress, when the strain reaches the plastic range, the moment will tend to fall off and the diagram will approach the form, $a' b' c' d'$ (Fig. 6(*d*)). The elastic curve tends to become flat; the curvature (and the region of over-strain) is confined to a short distance at each end (*p* and *q*, Fig. 6(*c*)).

There is precedent for ignoring over-stress of this type when strictly localized. Mention has been made of higher unit stresses allowed in bearing on rivets and pins (see "Introduction"). It may also be noted that most riveted end connections are productive of considerable local over-stress.

Reference may also be made to the fact that the presence of a hole in a plate, otherwise uniformly stressed, results in a heavy stress concentration at the edge of the hole. If the diameter of the hole is so small that the width of the plate may be assumed infinite in comparison, analysis shows that the stress at the edge of the hole is three times the average. For normal ratios of diameter to widths, tests have shown stresses 2.3 times the average. However, the re-adjustment that occurs when the highly stressed region reaches the yield point, practically nullifies any effect on the ultimate strength. The two stress graphs are shown in Fig. 7. Below the elastic limit relatively high local stress concentrations must occur in all plates composing riveted members; but, for reasons just stated, these are properly ignored in all normal cases.[9]

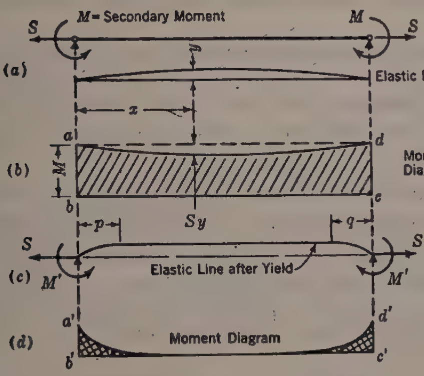

FIG. 6.

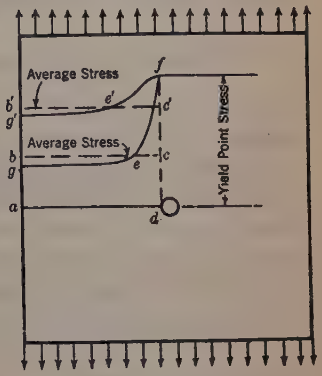

FIG. 7.

If the ultimate strength of a tension bar is understood to be its extreme effective strength as a bridge member, this value is limited to an average stress, $\frac{S}{A}$, equal to the yield point of the material. The preceding discussion tends to show that this effective strength is unchanged by any degree of secondary bending that may be expected in a well-designed structure.

Compression Members.—It is well known that flexure combined with compression produces a different behavior in a member from that which occurs under flexure and tension. The axial stress in the latter case tends to decrease the bending where in the former case it increases it; the moment represented by the axial stress times the deflection adds to the normal flexural moment, and in moderately flexible members at high stresses the total moment builds up rapidly. When the more highly stressed fibers pass the proportional limit, and the deflection begins to increase faster than the stress, the member proceeds rapidly to failure.

Most of the compressive chords (and frequently other compression members) in a bridge truss are of a stocky type which, even when subjected to bending stresses of from 50 to 100% of the axial stresses, shows relatively slight deflections from a straight line. The ultimate strength of such members is reached when the average stress over the section reaches, or approaches, the yield point of the material.

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[9] Reference may be made to "Drang and Zwang", von A. and L. Föppl, pp. 303–305, for presentation of the theory (originally due to Kirsch); see, also, "Applied Elasticity", by Timoshenko and Lessells, p. 9; and "Theorie und Berechnungen der Eisernen Brücken", von F. Bleich, pp. 249–252.

In considering the strength of compression members under primary and secondary stresses, the secondary end moments may be either of the same or of opposite signs. When the moments are of the same sign and of nearly the same magnitude, the member is bent into an S-shaped curve, the moment near the center is zero, and the deflections are so small that in any ordinary case buckling action for the member as a whole (giving rise to an $\frac{L}{r}$ failure) is practically negligible. When the moments are oppositely directed, the member is bent in single curvature, the maximum deflection occurs near the center, and the buckling tendency may be considerable.

Compression Member Bent in Single Curvature.—For simplicity consider the case in which the end moments are equal in magnitude. One analysis of the effect of secondary bending on the ultimate strength of a column bent in single curvature proceeds on the assumption that the action is analogous to the case of a virtual eccentricity $= \frac{M}{S}$ at each end.[10] By this method it is found that a considerable reduction in column strength results from the secondary bending. For example, 40% secondary stress reduces the strength of a short column about 28%, assuming the limit to be determined by the maximum fiber stress, $\frac{S}{A} + \frac{Mc}{I}$. A column with a slenderness ratio, $\frac{L}{r} = 75$, however, is reduced 35%, due to the added buckling tendency from the end moments.

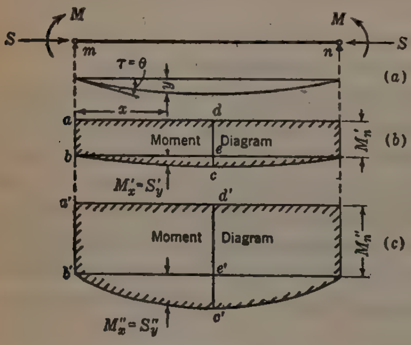

FIG. 8.

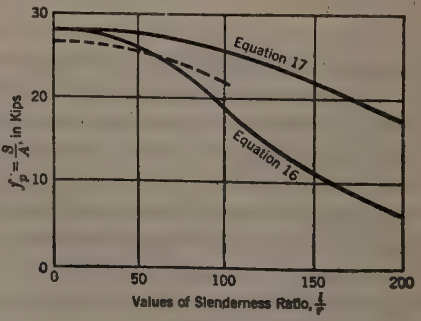

FIG. 9.

This theory is open to serious criticism as applied to members that develop any considerable curvature of the central line before failure, because, in such cases, the effect of the moment, $S\,y$, is such as to cause the angle, τ, to increase more rapidly than the loading. This effect is ignored in the common (as distinguished from the exact) theory of secondary stresses.

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[10] See "Modern Framed Structures", by Johnson, Bryan, and Turneaure, Pt. II, pp. 57–59.

Moment diagrams for two successive loading stages are shown in Figs. 8(*b*) and 8(*c*). It is clear that: (1) $\frac{\tau 1}{\tau 2} = \frac{\text{Area } abcd}{\text{Area } a'b'c'd'}$; and (2), that these ratios, due to the effect of the moment, $S\,y$, cannot be in the proportion of $\frac{M'_n}{M''_n}$. A more thorough analysis presented elsewhere[11] shows that if $q = \sqrt{\frac{S}{EI}}$, $\phi = \frac{qL}{2}$, and α = percentage of secondary stress,

$$f_b = \alpha\, f_p \frac{\phi}{\sin \phi} \quad \text{.......................} (7)$$

This remarkably simple form admits of a ready comparison with the case in which $M = S\,e$; thus:

$$f_b = \frac{e\,c}{r^2} f_p \sec \phi \quad \text{.......................} (8)$$

from which is derived the ordinary "secant" formula for columns:

$$f_p = \frac{f}{1 + \frac{e\,c}{r^2} \sec \phi} \quad \text{.......................} (9)$$

Similarly, by means of Equation (7):

$$f_p = \frac{f}{1 + \frac{\alpha\,\phi}{\sin \phi}} \quad \text{.......................} (10)$$

It will be found that, since $\frac{\phi}{\sin \phi}$ increases much more slowly than $\sec \phi$, the effect of secondary bending on the buckling of a column is relatively much less than for end moments that increase in direct proportion to the load (as would be the case for eccentric loading). Since $\alpha = \frac{f_b}{f_p}$ in the first case, and $\frac{e\,c}{r^2} = \frac{f_b}{f_p}$ in the latter case, if the same values are taken for these terms (say, α = 25% and f = 36 000) the curves[12] for Equations (9) and (10) will be as shown in Fig. 9.

This analysis is not strictly correct in that, while it seeks to make the tangential angle, and not the end moment, proportional to the load, it assumes the correctness of Equation (3). The correct relation, however, is:[13]

$$M_{mn} = \frac{2\,EI}{L}\,(2\,a\,\tau_{mn} + b\,\tau_{nm}) \quad \text{................} (11)$$

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[11] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 1221 *et seq.*

[12] *Loc. cit.*, p. 1223.

[13] "Modern Framed Structures", by Johnson, Bryan, and Turneaure, Pt. II, p. 512.

in which, a and b are converging infinite series in $q\,L$. Thus,

$$a = 1 + \frac{(q\,L)^2}{30} + \frac{11}{25\,000}\,(qL)^4 + \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

A rigorous analysis of the problem is possible, but the resulting equations are exceedingly involved and are not readily applied. Their interest is largely academic, since a glance at the graph of the approximate equation in Fig. 9 will show that, for all values of $\frac{L}{r}$ less than 70, the effect of secondary stress on buckling action, measured in this case by the ratio, $\dfrac{1 + 0.25}{1 + \dfrac{0.25\,\phi}{\sin\phi}}$, is slight, and for values less than 50, it is quite negligible. This holds true regardless of the value of α (herein assumed as equal to 25 per cent). Two further facts are to be noted:

(1) Extremely high secondary stresses nearly always occur in members bent in an S-curve. The lower section of the end post in sub-paneled trusses, or trusses with a collision strut, is practically the only exception to this rule.

(2) From a consideration of the properties of bridge trusses and the manner in which secondary stresses arise, it is virtually impossible for a high secondary stress to be developed in a member bent in single curvature which, at the same time, is slender enough to develop any considerable buckling action. The exceptions to this rule are too few to be of importance in ordinary design. With a percentage of secondary stress of 35, or less, the dotted curve on Fig. 9 shows that, as far as buckling is concerned, columns with values of $\frac{L}{r}$ ranging from 50 to 80, have practically the same strength as similar pin-ended columns with $\frac{e\,c}{r^2} = 0.25$.

Compression Members Bent in Double Curvature.—When the secondary end moments are in the same direction, the bar will be bent into an S-curve for which the moment diagram is as shown in Fig. 10. (It is again assumed for simplicity that the moments are equal.)

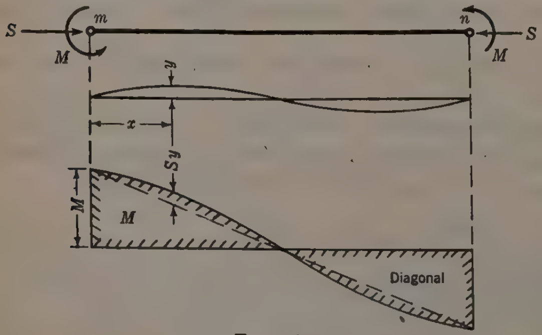

FIG. 10.

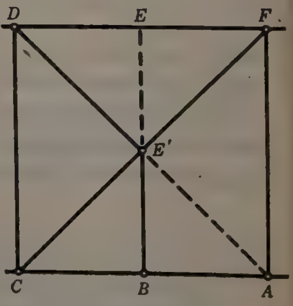

FIG. 11.

The deflections in this case will be much smaller than for bending in single curvature, and the points of maximum deflection will fall outside the quarter-points. Obviously, in such a case, the effect of secondary bending on the buckling action of the column as a whole is distinctly favorable since it forces the member to bend in double curvature, and the added bending moment due to axial stress is practically negligible. The high flexural stresses are near the ends of the member, and, in general, the effect of the secondary stresses on ultimate strength is the same as for tension members previously discussed; that is, a high extreme fiber stress occurs over a limited region, and in the neighborhood of the yield point the re-adjustment of stress due to plastic flow reduces the effect to negligible proportions before the average stress reaches the yield point.

Local Buckling in Thin-Walled Compression Members.—Most bridge trusses of the massive type for which the secondary stresses reach high percentages will have relatively stocky chord members $\left(\frac{L}{r} \leqq 40\right)$. Built-up members of such proportions show little or no transverse deflection until the average unit stress passes the proportional limit, and their ultimate strength is ordinarily from 80 to 100% of the yield-point stress of the material. It is generally considered that, for columns of this type, failure is imminent when any considerable part of the section is stressed to the yield point (although the average stress may be considerably less). More often than not the failure occurs through the local buckling of one of the comparatively wide, thin plates that make up the section.

Consider the top chord of a bridge of the conventional box type of section. Assume the material to have a yield point of approximately 35 000 lb per sq in. and the maximum permissible primary unit stress to be 17 000 lb per sq in. The secondary stress will be assumed as 100% of the primary. Under maximum working loads, then, the material on the most stressed side is loaded approximately to the yield point. If the bridge should be subjected to a 50% over-load, and the secondary moments are taken as proportional to the angular changes (that is, if Equation (3) were still to hold), the extreme fiber stress would be 51 000 lb per sq in., although the average would be only one-half this value. It has already been shown that beyond the proportional limit important re-adjustment of stress and strain takes place, resulting in a large percentage decrease in the actual secondary stress. None the less, if the member is assumed to be bent so as to throw the upper side into compression, the entire cover-plate and a portion of the top angles and web (amounting perhaps to 25% of the entire section) will be stressed to the neighborhood of the yield point. It would appear reasonable to suppose that the cover-plate, considered as an independent piece, would be on the verge of buckling as soon as the yield point is reached, and since the high secondary bending causes this state of stress to be reached much sooner than otherwise, it is pertinent to ask whether this may not hasten failure and thus reduce the ultimate strength of the member.

The analysis of the buckling of plates due to loads in their own plane is now well developed. The theory was initiated by Professor G. H. Bryan[14] many years ago; it has been further developed and its applications (particularly to structural problems) greatly extended by Professor S. Timoshenko[15] in his classic memoir on elastic stability. Further important contributions to the subject have been made by Professor H. Reissner,[16] the late Hans H. Rode,[17] M. Am. Soc. C. E., and H. M. Westergaard,[18] M. Am. Soc. C. E.

The most comprehensive analysis of the buckling of plates as parts of bridge members that has yet appeared, so far as the writers know, has been presented by Professor F. Bleich.[19] Professor Bleich attempts to include in his analysis the effects of partial restraint at the edges of the plate and also to extend the results into the "plastic" range.

Some results of these analyses may be summarized briefly. If a plate of length, a, width, b, and thickness, t, is considered to be loaded in the direction of a, with a uniformly distributed compression, $P = f_p\, t\, b$, and restrained against linear (but not angular) displacement along the edges parallel to the loading, Bryan's formula for buckling in a single wave is,

$$f_p = \frac{\pi^2 E}{12\,(1 - m^2)} \left(\frac{t}{b}\right)^2 \left[\frac{a}{b} + \frac{b}{a}\right]^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

in which, m = Poisson's ratio (usually 0.25 to 0.3 for steel).

Bleich proposes using a variable factor, τ, a function of the stress, f_p, to modify the value of E for stresses beyond the proportional limit. For conditions assumed in Equation (13), he derives,

$$f_p = \frac{\pi^2 E \sqrt{\tau}}{12\,(1 - m^2)} \left(\frac{t}{b}\right)^2 \left[\frac{a}{b\,\sqrt[4]{\tau}} + \frac{b\,\sqrt[4]{\tau}}{a}\right]^2 \quad\ldots\ldots\ldots(14)$$

For $\tau = 1$, this reduces to Bryan's formula.

From Equation (13), for a fixed ratio of $\frac{t}{b}$, f_p will vary with the quantity in the square bracket and will be a maximum for $a = b$, giving,

$$f_p = \frac{\pi^2 E}{3\,(1 - m^2)} \left(\frac{t}{b}\right)^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

Taking $m = 0.3$, Equation (15) gives 67 900 lb per sq in., and 43 400 lb per sq in., respectively, for $\frac{t}{b} = \frac{1}{40}$ and $\frac{t}{b} = \frac{1}{50}$

If the edges are fully fixed, the foregoing buckling loads are practically doubled. Undoubtedly, the true condition is intermediate between these extremes. These results tend to show that for normal proportions of a compression chord, buckling of the cover-plate will not take place within the elastic limit of the material. Beyond this point, of course, the formula does not apply.

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[14] *Proceedings*, London Math. Soc., 1891, p. 54.

[15] "Sur la Stabilité des Systémes Elastiques", *Annales des Ponts et Chaussées*, 1913, Vol. 3, p. 496 *et seq.*

[16] "Über die Knicksicherkeit ebener Bleche", *Zentralblatt der Bauverwaltung*, 1909, p. 93.

[17] 'Beitrag zur Theorie der Knickerscheinungen", *Der Eisenbau*, 1916, p. 281 *et seq.*

[18] "Buckling of Elastic Structures", *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

[19] "Theorie und Berechnungen der Eisernen Brücken", pp. 216–239, Berlin, Julius Springer, 1924.

In evaluating his expression for the buckling load in the more general case in which some of the material may be strained beyond the proportional limit, Bleich, after a lengthy analysis, obtains for f_p (in tons per square centimeter),

$$f_p = \frac{\phi}{2} - \sqrt{\frac{\phi^2}{4} - 9.61} \quad \text{.........................(16)}$$

In which, $\phi = \dfrac{\left(\dfrac{b}{t}\right)^4}{E \times 10^4} + 6.2.$

For $\dfrac{t}{b} = \dfrac{1}{40}$, f_p = 36 600 lb per sq in.; and, for $\dfrac{t}{b} = \dfrac{1}{50}$, f_p = 32 800 lb per sq in. Again, it should be noted that these values are for the case of no edge restraint.

Any attempt to take into account the variation in E beyond the proportional limit is attended with much difficulty; hence Equation (16) cannot be regarded as more than roughly approximate, and is scarcely applicable in the region of large plastic flow.

In applying the buckling formulas to the secondary stress problem, consideration must be given to the important difference between a plate with freely supported edges, loaded in its plane, and, say, a cover-plate of a chord member under combined primary and secondary stress. In the former case, when the stress reaches, or closely approaches, that corresponding to the point, B, on the stress-strain curve of Fig. 2, any further increase will result in distortions many times as great as those previously sustained. Such distortion has the effect of grossly exaggerating any slight imperfection in material or manufacture, and since no plate is perfectly straight or homogeneous, relatively large transverse deflections are developed which lead to immediate collapse.

In the case of a cover-plate subjected to combined primary and secondary stress causing the extreme fiber to reach the yield-point value, it is equally true that any material increase in the stress will probably result in a buckling failure. As has been shown, however, this increase cannot take place as long as the average stress on the section is well below the yield point, since it must be accompanied by a large plastic flow, and, due to the restraint of the adjacent material, such large flow cannot possibly occur in one part of the section until practically the entire area is stressed to the yield-point region. It appears reasonable to suppose that what actually happens is that when the yield-point stress is reached in the cover-plate, a practically constant state of stress is maintained under increasing load, while the deformation (which in this region of large plastic flow may vary widely with almost no change in the stress) varies only as is required to produce the necessary secondary displacements. As long as the average primary stresses are within the proportional limit, these secondary displacements will increase no faster than the applied loads. In other words, the free flow (accompanying a stress reaching the yield point), which would take place in an independent plate, is restrained to comparatively small limits by the action of the neighboring material in the case of axia stress and secondary bending, and this, it may be expected, will have a larg effect in reducing the buckling tendency in the cover-plate.

When the average stress over the section passes the proportional limit, the cover-plate distortion begins to increase rapidly, and somewhere between this and the yield point of the column as a whole, local buckling of the cover may be anticipated.

One further point deserves some emphasis. When the common theory of secondary stresses is applied, it is assumed, of course, that the law of linear elasticity applies and that the effect of bending due to $S\,y$ is negligible. When, therefore, as discussed previously, an average primary stress of 26 000 lb per sq in., and a secondary stress equal to 100% of the primary, are computed for any member, what really happens is that the member is distorted at the end so that the tangential angle is such as would correspond to 26 000 lb per sq in. flexural extreme fiber stress, assuming E to be constant and the effect of axial stress on bending negligibly small. If the stress-strain diagram of Fig. 2(a), is examined, however, it will be noted after the limit of strict proportionality is passed (that is, when E becomes variable and a function of the load), that there is a considerable deviation from linearity before the region of free plastic flow is reached. The actual deformation at Point B (the beginning of marked plastic flow) will ordinarily be equivalent to that which would be produced by a stress of 40 000 to 50 000 lb per sq in., if the material followed the linear stress-strain law. In other words, the variation in E, before the actual yield point is reached, is sufficient to take care of a nominal extreme fiber stress due to combined primary and secondary effects considerably greater than the yield point of the material, and probably equal to any value to be found in well-designed trusses, even of the heaviest and most rigid type.

The possible effect of stresses beyond the elastic limit upon local buckling, especially of the cover-plates in the standard type chord sections, is the one point upon which the deductions from rational analysis are rather indecisive, and, if the preceding analysis is sound, the only manner in which there is any reason to suspect that the ultimate strength of a compression member may be affected appreciably by secondary stress. These facts seemed to justify an experimental study of the phenomena of failure in a built compression member of the box type subjected to a high percentage of secondary stress, with particular emphasis on the behavior of the cover-plate in the region of high local stress. In the following section, an account is presented of a limited group of laboratory tests undertaken to throw some light on this particular phase of the secondary stress problem.

EXPERIMENTAL STUDY

Model Simulation of Conditions.—The experimental study consisted of the testing of four columns of symmetrical section of standard types, giving in all eight separate tests of columns under the action of secondary stress and direct axial, or primary, stress. Column No. 1 was made up in the shop of the Experimental Engineering Laboratory, University of Minnesota, Minneapolis, Minn., and the remaining sections were fabricated in the Minneapolis Plant of the American Bridge Company. All specimens were of relatively small cross-section as the testing machine available was not of sufficient capacity to handle a full-sized normal bridge chord.

Because of the difficulty of simulating the partial fixity of the ends of a member in a riveted truss, introducing as it would the problem of controlling and measuring the induced rotations, all specimens were tested with hinged ends. Such a member is closely analogous to the top chord member in a pin connected sub-paneled truss with the members continuous over the sub-panel points (see Fig. 11). A truss of this type, while nominally classed as pin-connected, may show high secondary stresses at such places as Point *E*. However, it should be emphasized again, the object of the experiments was to study the behavior of built compression members under high local stress due to combined primary and secondary action. As long as the essential features of such action are maintained, it is immaterial for the purposes of the test whether or not all details correspond to those which obtain in the case of an actual truss member under high secondary stress.

It is true, of course, that ordinarily the highest percentages of secondary stress occur in trusses with fully riveted chords. In Fig. 11, Joint *E′* will deflect about the same amount regardless of the end conditions of Member *D-F*. If the end joints are fully riveted, the added stiffness of the member due to the partial restraints at Points *D* and *F* will cause (for approximately constant center deflection) considerably higher secondary stress at Point *E* than would be the case for pin connections. As long as the fundamental characteristics of secondary stress behavior are preserved, however, high stresses, artificially produced, will have the same local effect on a member of given cross-section as if these same stresses were brought about by exact simulation of all the details of riveted truss action. This remark applies also to the method of loading which was followed in the test.

Again referring to Fig. 11, it will be noted that if Member *EE′* is removed, there will be no secondary stress at Point *E,* since this latter is caused by the displacement of Point *E′* relative to Line *DF*. This displacement tends to be transmitted to Point *E,* through Member *EE′* (the blank strut), developing a considerable tension in the latter. The action is identically that of a beam, *DEF*, with a transverse load at Point *E*, with this exception: The deflection of Point *E′* (and, therefore, of Point *E*) is strictly limited by the relative displacement arising from axial distortions, and which may be computed analytically or by means of a Williot diagram. Quite contrary to the case of a free transverse load at Point *E*, when the displacements have reached the prescribed value, Point *E* is rigidly held in position, no further deflection being possible. This is the unique characteristic of secondary stress action. Clearly, it can be simulated accurately in the case of a column by applying the axial loads, together with a certain controlled deflection. If, in such case, one wishes to study the behavior of the member at a given stage of combined primary and secondary stress, it is immaterial whether (*a*) a given primary stress is applied, and then the desired deflection is induced; (*b*) the desired deflection is induced and then the direct stress is applied; or (*c*), the two are developed simultaneously. The second method was followed in this investigation.

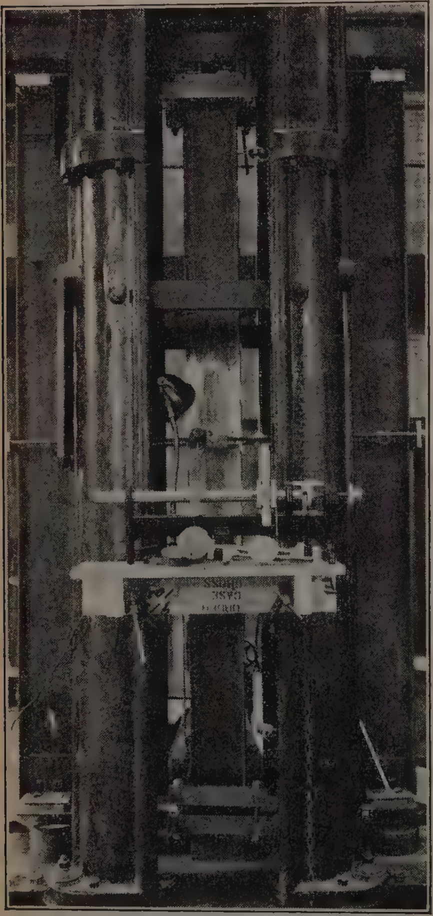

FIG. 12.—SET-UP OF EXPERIMENTS ON SECONDARY STRESSES.

The columns tested were made to conform to the foregoing condition by being subjected to a constant bending deflection and consequent secondary stress as the axial stress was increased. This combination gave maximum effects due to combined secondary and primary stresses at the center line, with primary stresses only acting at the ends. In order to realize this condition the columns tested were placed inside a rectangular frame of rigid I-beam sides with thin flexible ends. The bending deflection of the column was then maintained at a constant value by the use of a toggle extending about the sides of the frame. The flexible end plates were sufficient to transfer the transverse thrust, but not stiff enough to cause appreciable moment on the end of the member or to interfere with the axial loading of the column by the testing machine. In this manner a combination of stresses was maintained in the cover-plate of the column analogous to the combined primary and secondary stresses at the stay connection of Member *DEF* in Fig. 11. The actual amount of the secondary stress was thus dependent upon the transverse deflection given the specimen and was varied somewhat for different sections, depending upon their properties. Reference to the general photograph of the set-up (Fig. 12), and to the detail drawing of Column No. 2 (Fig. 13), will help to clarify the description of the specimen and testing frame.

Column Dimensions and Properties.—Column No. 1, Table 1, was similar to the standard top chord section of a bridge, consisting of plates and angles with a top cover and open bottom (see Fig. 14).[20] Because of the limited capacity of the testing machine and the difficulty of providing the necessary transverse bending force in the toggle, the model was made one-fifth the area

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[20] The proportions follow closely the top chord of the typical truss given in "Modern Framed Structures", by Johnson, Bryan, and Turneaure, Pt. III, Pl. III.

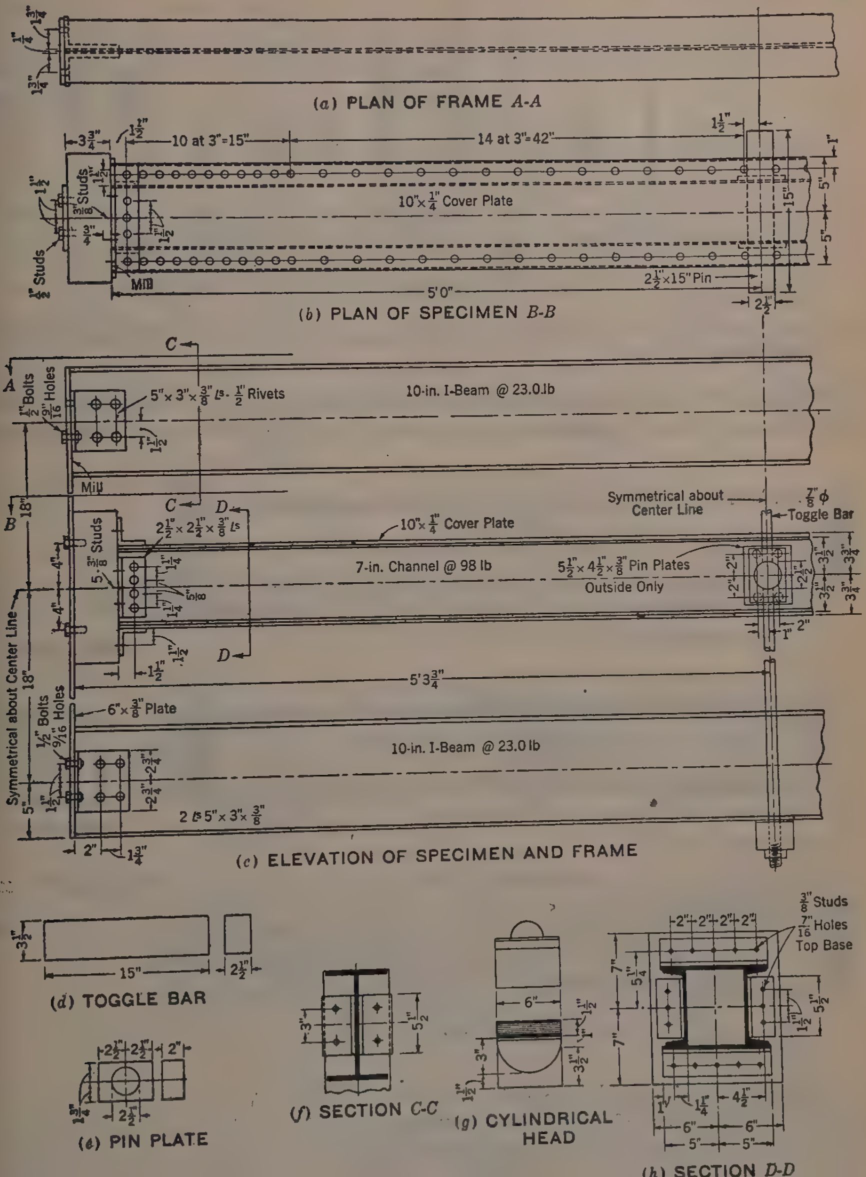


FIG. 13.—DETAILS AND DIMENSIONS OF COLUMN NO. 2.

of the chord member referred to, as closely as commercial material would allow. (The diameter of the rivets was 0.17 in., the rivet holes being drilled. The rivet spacing in the angles is the same as in the cover-plate.) For Test

TABLE 1.—CHARACTERISTICS OF TEST COLUMNS

Column No.	Test	Area, A, in square inches	Length, L, in inches	Moment of inertia, I, in inches*	Distance, c, from neutral axis to extreme fiber, in inches	Radius of gyration, r, in inches $= \sqrt{\frac{I}{A}}$	Slenderness ratio, $\frac{L}{r}$	Thickness, t, of cover-plates, in inches	Distance, b, between center lines of rivet rows, in inches	Ratio, $\frac{b}{t}$	Thickness, w, of web, in inches	Rivet spacing, p, in inches at center line	Diameter of rivets, D, in inches
1	1	3.91	86.3	23.5	3.04	2.45	35.2	⅛	5⅛	41	⅛	2	0.17
	2	.....	80.6				33.0						
2	.....	10.70	128.2	108.2	3.75	3.18	40.3	¼	8	32	0.24	3	⅜
3	.....	9.82	128.2	91.2	3.69	3.13	41.0	$\frac{3}{16}$	9	48	0.24	2¼	⅜
4	End	9.82	.....	.....	......	....	....	....	....	....	....	.....	.....
	Center	.....	128.2	91.2	3.69	3.13	41.0	$\frac{3}{16}$	9	48*	0.24	2¼	⅜
	Line	10.99	.....	.....	......	....	....	....	....	....	....	.....	.....

* Reinforced.

2, Column No. 1 was the same as for Test 1, except that it was shorter. The only change in the riveting was in the cover (see Fig. 14(d)) for which a new

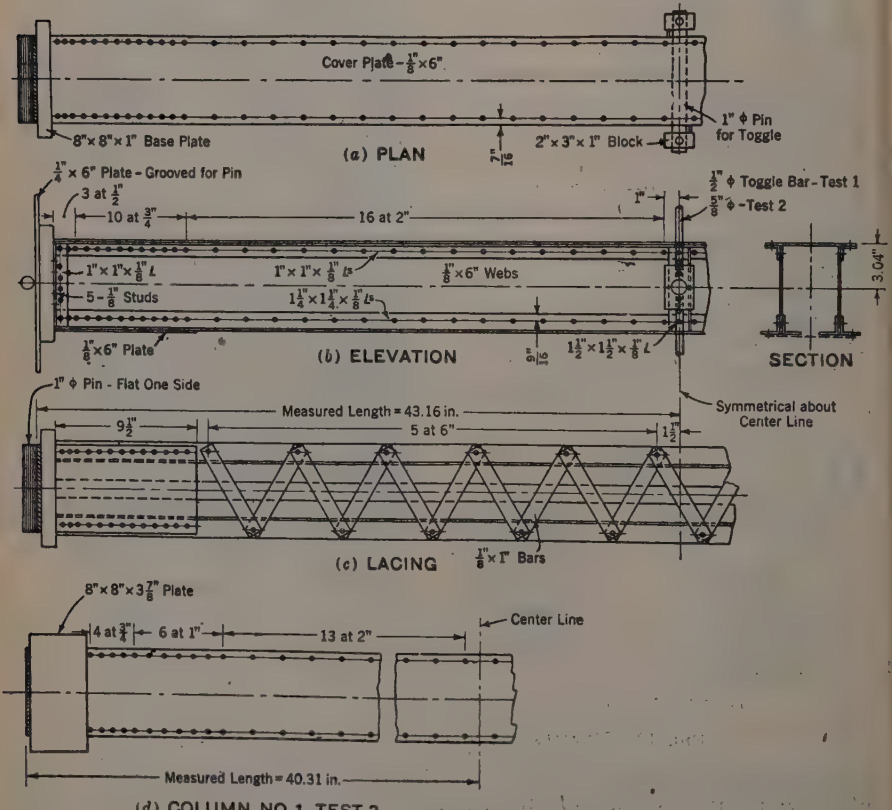


FIG. 14.—DETAILS AND DIMENSIONS, COLUMN NO. 1.

base plate was provided; otherwise, the details at the center line and ends were the same in both cases. Columns Nos. 2, 3, and 4 were of slightly different section, being made of two standard channels with two cover-plates (closed). Because of the symmetry of the section, two tests could be made from each specimen. They were fabricated in the standard manner using punched holes and $\frac{3}{8}$-in., hot-driven rivets, while Column No. 1 had small cold-driven rivets in drilled holes. The ends of the members were milled to provide full bearing. The properties and detail drawings of the various column sections are shown in Figs. 13 to 16. Column No. 4 was the same as Column No. 3 (Fig. 15) except for the welded bars shown in Fig. 16.

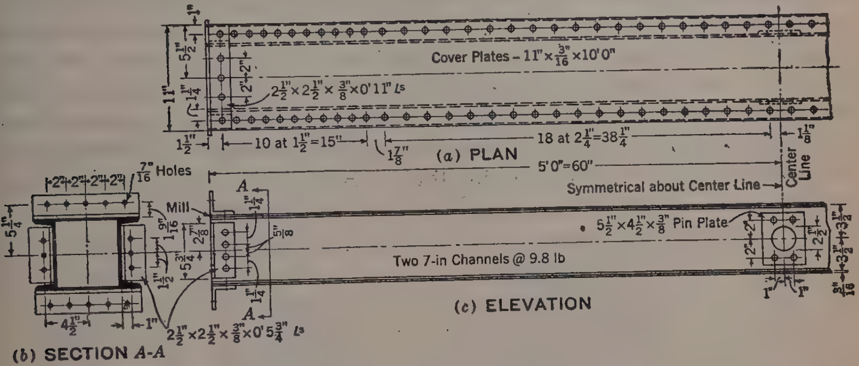


FIG. 15.—DETAILS AND DIMENSIONS, COLUMN NO. 3.

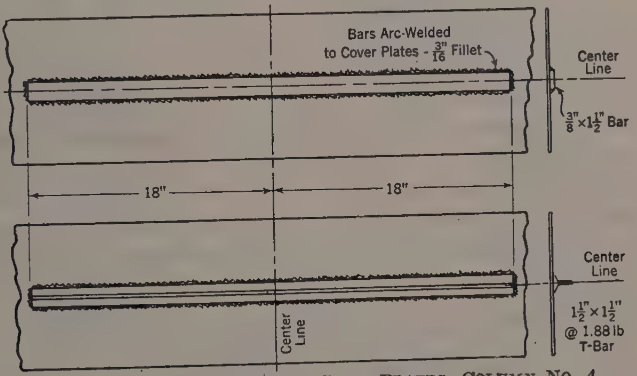
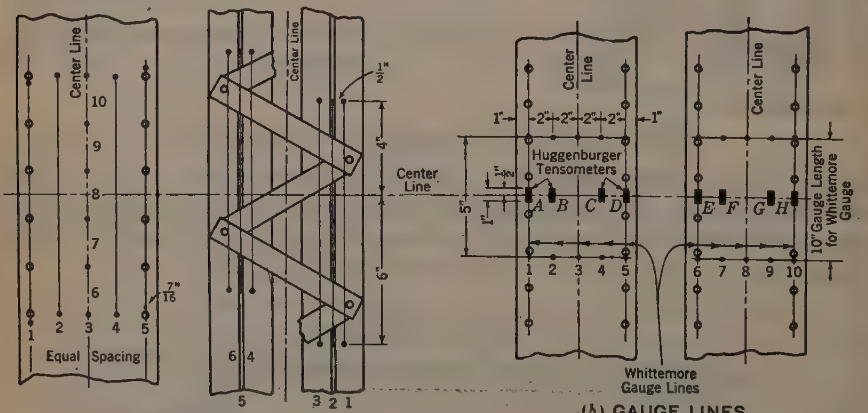

Bars Arc-Welded to Cover Plates - 3/16" Fillet
Center Line
3/8" × 1½" Bar
18"
18"
Center Line
Center Line
1½" × 1½" @ 1.88 lb T-Bar

FIG. 16.—BARS WELDED TO COVER-PLATES, COLUMN NO. 4.

Test Procedure.—The axis of the member was set as nearly vertical in the testing machine as could be done with a hair wire and weight. The pins were placed at the centroid of the section as closely as this could be measured and transferred to the base plate. Pins were flattened on one side to fix the point of rotation and to give a definite value of L in the plane considered. They were supported on cylindrical heads in the other plane to fit them to any angularity between the column ends and the testing machine. The detail drawing of Column No. 2 (Fig. 13) will make the test set-up clear.

The initial loading used in all cases was 1 000 lb and the increments of axial loading were equivalent to 2 500 lb per sq in. on the section for the first test, and 5 000 lb per sq in. for the second test, on each specimen. Strain readings were taken at all increments of loading and, in many cases, loadings were repeated several times to establish the behavior of the member more definitely. Strains were obtained on Column No. 1 by the Whittemore strain-gauge, and on Columns Nos. 2, 3, and 4 by both Huggenberger tensometers and the Whittemore gauge. Consistent deformations were obtained to 0.00001 in. per in., or equivalent stresses of approximately 300 lb per sq in., with the aforementioned apparatus. Gauge lines on Column No. 1 consisted of five 10-in. lines on the compressive face, and six 10-in. lines on the tensile face placed symmetrically about the center line. Gauge lines on Columns Nos. 2, 3, and 4 consisted of one 10-in. line and four 1-in. lines on each face symmetrically placed. (See Fig. 17.)



(a) GAUGE LINES COLUMN 1

(b) GAUGE LINES COLUMNS 2, 3, & 4

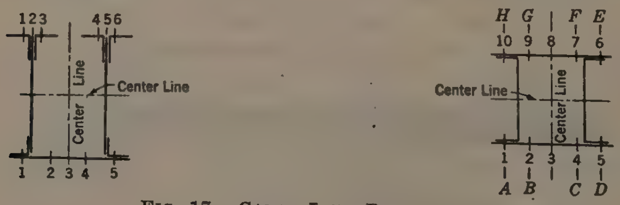


FIG. 17.—GAUGE LINE DIMENSIONS.

As the deflection of the column was to be maintained at a constant value such that the nominal secondary stress remained constant throughout the range of primary loadings below the material yield point, it was necessary to measure the deflection value accurately. This was accomplished by means of an optical micrometer in the case of Column No. 1 and an electrical contact screw micrometer for Columns Nos. 2, 3, and 4, measuring the deflection between a hair wire and a point on the specimen center line in the web. It must be noted that the actual column deflection was not measured in any case, as the hair wire was not fastened to the point of rotation of the column ends, but to some point on the deflecting structure. This had no effect on the problem because it was only necessary to have a relative value that could be measured and maintained. The deflection was not used directly for stress calculations, but was kept at a constant relative value in order to give a certain nominal secondary stress. In the case of Columns Nos. 2, 3 and 4, the wire was fastened to the one-tenth points on the specimen. This fastening the actual deflection values were read, while in the case of Column No. 1 the wire was fastened to the one-tenth-points on the specimen. This fastening necessitated a calculation of the elastic curve of the column so that an approximate value of the relative deflection between the one-tenth points and the center would be available. The elastic curve was calculated carefully, and it was noted that little change occurred in the deflection values for a large range of axial loadings. The equation for the elastic curve based on a constant center deflection and variable axial loading takes the following form:

$$y = \frac{\Delta}{\dfrac{\tan\left(\dfrac{qL}{2}\right)}{q} - \dfrac{L}{2}} \left[\frac{1}{q} \, \frac{\sin(q\,x)}{\cos\left(\dfrac{qL}{2}\right)} - x \right] \text{..........(17)}$$

in which, Δ = center deflection and $q = \sqrt{\dfrac{S}{EI}}$.

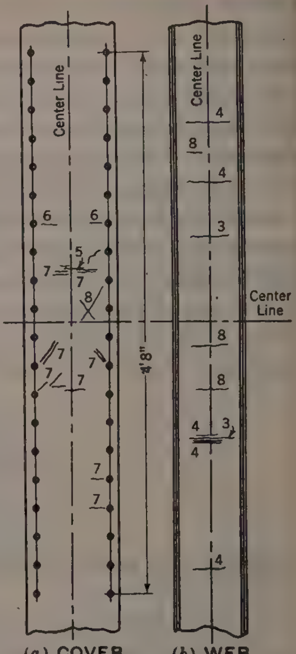
It was decided to use a secondary stress of approximately 13 000 lb per sq in. for Column No. 1, Test 1, giving roughly 50% secondary stress at the material yield point. Since this test indicated that such a value was insufficient to cause local failure to develop before the normal ultimate strength of the columns was reached, in all later tests a value of 20 000 lb per sq in. was used in an attempt to produce a more marked local effect. Using approximately a secondary stress of 20 000 lb per sq in., the required deflection of the column was calculated and this value used as a basis for the approximate measurement in the tests. Table 2 gives calculated deflections and lateral loads corresponding to required secondary stress values. (The action of the column under lateral load was that of a simple beam prior to the application of axial loadings.)

TABLE 2.—CALCULATED LATERAL LOADS AND DEFLECTIONS FOR ASSUMED SECONDARY STRESS

Column No.	Lateral load, P, in pounds	Displacement, Δ, in inches	Stress, f', in pounds per square inch
1......................	4 750	0.0900	13 250
2......................	19 200	0.2685	21 300
3......................	16 200	0.2556	20 000
4......................	16 200+	0.2556	20 000

During the testing operations the deflection was checked and corrected if necessary at each increment of axial loading, and when the axial stress reached a value sufficient to reduce appreciably the lateral force necessary to maintain the deflection, the stability of the columns was assured by stressing the toggle on the compressive face, thereby preventing sudden $\frac{L}{r}$-failure in the direction of the imposed deflection. This additional toggle was also used in adjusting the deflection after the bending resistance of the column had been reduced due to plastic flow and the original transverse force would have caused more than the required deflection.

Before testing, the specimen was painted with a dilute solution of plaster of Paris and water to make the strain lines visible. These are clearly shown in Fig. 19. The strain lines were recorded and sketched as they appeared during the test and a typical set is shown in Fig 18, with Table 3. Determination of the transverse force necessary to maintain the required deflection was accomplished by strain measurements on the toggle bars and subsequent calculation of the force transmitted. As an indication of the reduction in moment and consequent reduction in the transverse force, *P*, as the material in the column cover-plate approached the yield point, and stress re-adjustment took place, the measured *P*-values are given in Table 4 for Columns Nos. 2 and 3.



(*a*) COVER PLATE (*b*) WEB

FIG. 18.

TABLE 3.—OBSERVATIONS OF STRAIN LINES, TESTS 3 AND 4

TEST 3		TEST 4	
Strain Line No. (see Fig. 18):	Load, in pounds	Strain Line No. (see Fig. 18):	Load, in pounds
1	161 500*	5	157 000
2	188 250*	6	215 000
3	301 000	7	268 500
4	339 000	8	322 000
....	....	....	354 000†

* Lines at Loads 1 and 2 for face not shown in Fig. 18.

† Innumerable lines appeared at this load.

For the purpose of this paper, it was not considered desirable to reproduce all the test data in detail. Tables 5 and 6 present the data from Column No. 1, Test 2, and Column No. 2, Test 3, showing deformations in millionths of an inch per inch for various axial loadings. These data are presented as typical examples of all eight tests, there being remarkably close agreement in all cases.

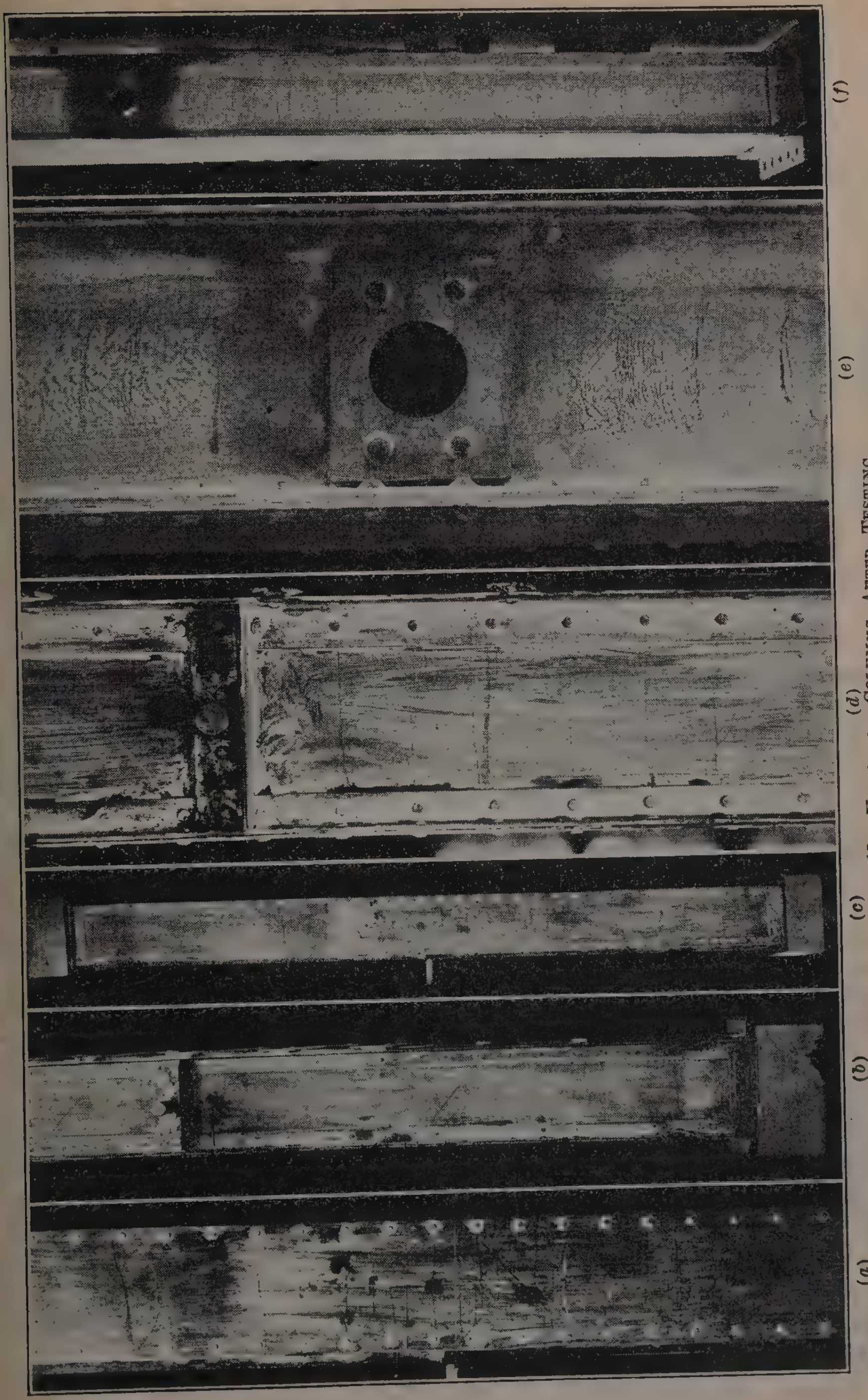


FIG. 19.—VIEWS OF COLUMNS AFTER TESTING.

TABLE 4.—OBSERVED VALUES OF LATERAL LOAD, P.
($E = 30.0 \times 10^6$)

Primary load, S, in pounds	LOADING RODS ($\frac{7}{8}$-INCH ROUND; $A = 0.60$ SQUARE INCH)			STAY-RODS ($\frac{5}{8}$-INCH ROUND, $A = 0.30$ SQUARE INCH)			Total load, P, in pounds
	Deformation, in millionths of an inch per inch	Stress, f', in pounds per square inch	Load, P, in pounds	Deformation, in millionths of an inch per inch	Stress, f', in pounds per square inch	Load, P, in pounds	
			(*a*) COLUMN NO. 2, TEST 3				
1 000*	564	16 900	20 300	....	....	....	20 300
54 500*	551	16 550	19 800	....	....	....	19 800
108 000*	538	16 150	19 400	....	....	....	19 400
161 500*	493	14 800	17 800	....	....	....	17 800
188 250*	497	14 900	17 900	171	5 130	3 080	14 820
215 000*†	495	14 850	17 800	264	7 930	4 760	13 040
			(*b*) COLUMN NO. 3, TEST 5				
1 000*	515	15 500	18 600	....	....	....	18 600
50 100*	....	....	....	18.5	550	330	18 270
123 800*	463.5	13 900	16 700	....	....	....	16 700
148 300*	....	....	....	40	1 200	720	15 980
197 400†	....	....	....	167	5 010	3 010	13 600

* Column is in deflected position.
† Readings discontinued at this point.

The ultimate loads for Tests 2 and 3 were 167 800 lb and 381 800 lb, respectively, corresponding to a stress of 42 800 lb per sq in. and 35 700 lb per sq in. In Column No. 1 (see Table 5), failure occurred first in the web, near the base, after which the cover-plate buckled. A set of 80 millionths of an in. per in, is shown on the tension side, which indicates that even this side was stressed beyond the elastic limit (compression forces).

TABLE 5.—OBSERVED DEFORMATIONS, COLUMN NO. 1 (TEST 2), IN MILLIONTHS OF AN INCH PER INCH, FOR VARIOUS AXIAL LOADINGS
(+ = Lengthening (Tension))

Loads, S, in pounds	READINGS AT GAUGE LINES (SEE FIG. 17):											No. of sets	AVERAGES	
	1	2	3	4	5	1	2	3	4	5	6		Cover-plate	Bottom angles
	Cover-Plate					Bottom Angles and Web								
1 000 to 140 000*	—1 112	—1 113	—1 139	—1 156	—1 164	—1 061	—1 094	—1 080	—1 112	—1 147	—1 132	2	....	....
1 000†	—624	—581	—590	—579	—586	+674	+831	+701	+672	+947	+700	2	—592	+686
40 000†	—973	—944	—936	—934	—941	+392	+557	+424	+393	+660	+412	2	—945	+405
80 000†	—1 298	—1 273	—1 272	—1 276	—1 285	+85	+239	+120	+60	+313	+74	2	—1 281	+85
100 000†	—1 728	—1 758	—1 800	—1 814	—1 721	—98	+55	—75	—115	+141	—101	2	—1 764	—97
120 000†	—2 666	—2 617	—2 819	—2 784	—2 662	—325	—174	—321	—359	—106	—344	2	—2 710	—337
140 000†	—3 530	—3 579	—3 713	—3 693	—3 427	—676	—559	—672	—728	—505	—718	1	—3 588	—690
1 000	....	....	....	....	....	—61	—7	—107	—133	....	—9	2	....	—89

* Column condition O. K. † Column is in deflected position.

Properties of the Column Material.—The modulus of elasticity of the material was determined from the load-strain relations of the column under axial loadings only, and checked in several cases by similar tests on coupons

TABLE 6.—OBSERVED DEFORMATIONS, COLUMN NO. 2 (TEST 3), IN MILLIONTHS OF AN INCH PER INCH, FOR VARIOUS AXIAL LOADINGS
(+ = Lengthening (Tension))

Load, S, in pounds	READINGS AT GAUGE LINES (SEE FIG. 17 (b)):										No. of sets	AVERAGES			
	3	*A*	*B*	*C*	*D*	8	*E*	*F*	*G*	*H*		*A* and *D*	*B* and *C*	*E* and *H*	*F* and *G*
	Compression Cover-Plate					Tension Cover-Plate									
1 000*	—743	—703	—755	—756	—709	+742	+687	+782	+842	+745	13	—706	—756	+716	+812
27 750*	—829	......	......	......	......	+665	......	......	......	......	4	......	......	......	......
54 500*	—915	—873	—917	—923	—863	+564	+527	+595	+650	+563	8	—868	—920	+545	+622
81 250*	—1 003	—958	—1 002	—1 008	—940	+458	+443	+502	+543	+468	2	—949	—1 005	+455	+522
108 000*	—1 104	—1 076	—1 122	—1 100	—1 008	+374	+359	+408	+446	+383	2	—1 042	—1 111	+371	+427
134 750*	—1 241	—1 170	—1 275	—1 175	—1 093	+269	+284	+306	+342	+297	2	—1 132	—1 225	+290	+324
161 500*	—1 436	—1 272	—1 267	—1 275	—1 195	+171	+200	+204	+243	+210	2	—1 233	—1 271	+205	+224
188 250*	—2 088	—1 316	—1 232	—1 200	—1 880	+ 11	+ 99	+ 60	+ 93	+ 81	2	—1 598	—1 216	+ 90	+ 76
215 000*	—2 557	—1 520	—1 745	—1 530	—4 560	—109	+ 15	— 51	— 12	+ 3	2	—3 040	(Buckle)	+ 9	— 31
251 000*	—3 796	—2 410	—2 475	......	......	—326	—136	—255	—233	—195	2	......	......	—166	—244
301 000*	—5 631	......	......	......	......	—671	—379	—578	—594	—479	2	......	......	—429	—583

* Column in deflected position.

cut from the column itself. Table 7 gives the moduli for each column as determined by the tests on the columns themselves. These values were used in determining stresses.

TABLE 7.—MODULI OF ELASTICITY, E, IN POUNDS PER SQUARE INCH, AS DETERMINED BY TESTS

Column No.	Instruments used	Range of loading stress, in pounds per square inch	Average modulus of elasticity	No. of sets
1.............	Whittemore.........	20 000	30.7×10^6	14
2.............	Whittemore......... Huggenberger.......	20 000 20 000	27.7×10^6 28.8×10^6	12 24
3.............	Whittemore......... Huggenberger.......	20 000 20 000	28.8×10^6 28.6×10^6	10 32
4.............	Whittemore......... Huggenberger.......	10 000 10 000	29.0×10^6 29.0×10^6	6 6

As it was necessary to conduct a part of the test with some of the material stressed beyond the elastic limit, the characteristic stress-strain curves for the material were obtained in order that an estimate of the stress increase after passing the elastic limit could be made. A piece of material from the same rolling was used to determine the relations for Column No. 1 (see Fig. 20, Curve *A*). For Columns Nos. 2, 3, and 4, a coupon was cut from the cover-plate of Column No. 2 (see Fig. 20, Curves *B* and *C*). The modulus of elasticity is seen to check closely that determined from the columns. It is noted that the curve is nearly horizontal beyond the proportional limit even though the deformation was several times that of the yield-point stress. On this basis the stress carried by any material beyond the yield point was assumed to be a constant and of intensity equal to the yield-point value.

An inspection of Fig. 20 shows that the increase in stress after the proportional limit is reached and before the curve becomes so flat as to make the stress nearly constant is approximately 3 500 lb per sq in. for Column No. 1 and from 2 500 to 3 000 lb per sq in. for the remaining columns. The proportional limit for the material of Column No. 1 was taken to be 39 000 lb per sq in., which gave a value of 42 500 lb per sq in. for maximum stress, a close check on the ultimate strength of the column. Similarly, the proportional limit of the materials of Columns Nos. 2 to 4, inclusive, was found to be 32 500 lb per sq in., with a maximum value of approximately 35 000 lb per sq in., which is also a good check on the ultimate strength of the columns.

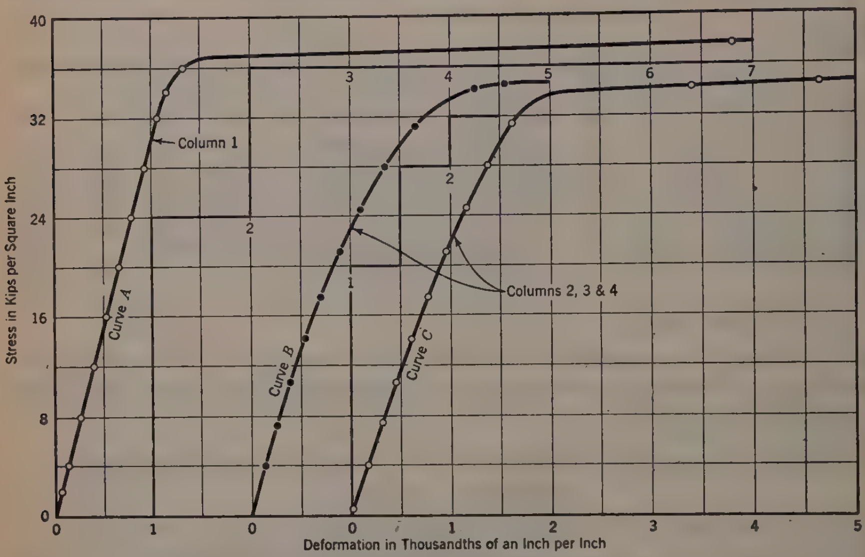


FIG. 20.—STRESS-STRAIN CURVES FOR MATERIAL IN TEST COLUMNS.

In computing stresses in the cover-plates from the measured strains, it must be remembered that the deformations obtained by the Whittemore gauge are average values over a 10-in. gauge length. (The Huggenberger gauges have 1-in. gauge lengths.) When the column is deflected under transverse moments these deformations do not represent the maximum strains, inasmuch as the strains due to bending vary from a maximum at mid-length to zero at the end of the column. This results in an appreciable change in the resultant unit deformation within the gauge length of the instrument when long gauge lengths are used. An approximation to the difference between the maximum and the average reading may be estimated on the basis of the moment change. Since the moment is nearly proportional to the distance from the end of the column (concentrated load), the moment diagram approximates a triangle and the deformation decrease is proportional to the average decrease in length from the end. In this case (L = 128.2 in. for Columns Nos. 2, 3, and 4), the percentage of change equals

$$\frac{\frac{5}{2} \times 100}{\frac{128.2}{2}} = 3.9.$$

Since the deformation from bending was approximately one-half the total, the error involved in neglecting this factor was approximately 2 per cent. As this value is approximately the probable error in reading, loading, and sectional area, it has been neglected in the curves and stress calculations.

TABLE 8.—STRESSES DETERMINED FROM STRAINS IN TABLE 5
(+ = Tension)

Load, *S*, in pounds	DEFORMATIONS, IN MILLIONTHS OF AN INCH PER INCH		STRESSES, IN KIPS PER SQUARE INCH	
	Cover-plate	Bottom angles	Cover-plate	Bottom angles
1 000*..........	— 592	+686	—18.2	+21.0
40 000*..........	— 945	+405	—29.0	+12.4
80 000*..........	—1 281	+ 85	—39.4	+ 2.6
100 000*..........	—1 764	— 97	—54.2	— 3.0
120 000*..........	—2 710	—337	—83.3	—10.3
140 000*..........	—3 588	—699	....	—21.5
167 800*..........	....	....	....	—40.0†
1 000..........	....	— 80	....	....

* Column in deflected position.
† Estimated from a set of 80 millionths of an inch per inch at 1 000 lb. with specimen not deflected.

Tables 8 and 9 give the stresses for the two tests as determined from the strains given in Tables 5 and 6 and the moduli of elasticity given in Table 7. After the proportional limit has been passed these stresses are incorrect, and the values are only given to show the nominal stresses that would occur in the plate were Hooke's law still operative. The actual values after the elastic limit has been passed, have been discussed previously.

TABLE 9.—DEFORMATIONS AND STRESSES DETERMINED FROM STRAINS IN TABLE 6
(+ = Tension)

Load, *S*, in pounds, with column in deflected position	WHITTEMORE GAUGE				HUGGENBERGER GAUGES; COVER-PLATE							
	Compression		Tension		Compression				Tension			
	Strain, in millionths of an inch per inch, Gauge Line 3	Stress, *f*, in kips per square inch	Strain, in millionths of an inch per inch, Gauge Line 8	Stress, *f*, in kips per square inch	Strain, in millionths of an inch per inch, Gauge Lines *A* and *D*	Stress, *f*, in kips per square inch	Strain, in millionths of an inch per inch, Gauge Lines *B* and *C*	Stress, *f*, in kips per square inch	Strain, in millionths of an inch per inch, Gauge Lines *E* and *H*	Stress, *f*, in kips per square inch	Strain, in millionths of an inch per inch, Gauge Lines *F* and *G*	Stress, *f*, in kips per square inch
1 000	— 743	—20.6	+742	+20.55	— 706	—20.3	— 756	—21.8	+716	+20.6	+812	+23.4
54 500	— 915	—25.3	+564	+15.6	— 868	—25.0	— 920	—26.5	+545	+15.7	+622	+17.9
81 250	—1 003	—27.8	+458	+12.7	— 949	—27.2	—1 005	—29.0	+455	+13.1	+522	+15.1
108 000	—1 104	—30.6	+374	+10.4	—1 042	—29.4	—1 111	—32.0	+371	+10.7	+427	+12.3
134 750	—1 241	—34.4	+269	+ 7.45	—1 132	—32.6	—1 225	—35.3	+290	+ 8.35	+324	+ 9.3
161 500	—1 436	—39.8	+171	+ 4.7	—1 233	—35.6	—1 271	—36.6	+205	+ 5.9	+224	+ 6.45
188 250	—2 088	—57.8	+ 11	+ 0.3	—1 598	—46.0	—1 216 (Buckle)	—35.0	+ 90	+ 2.6	+ 76	+ 2.2
215 000	—2 557	......	—109	— 3.0	—3 040	......	—1 637	......	+ 9	+ 0.26	— 31	— 0.89
251 000	—3 796	......	—326	— 9.0	......	......	—2 475	......	—166	— 4.78	—244	— 7.02
301 000	—5 631	......	—671	—18.6	......	......	......	......	—429	—12.4	—586	—20.4

Re-Adjustment in Stress-Strain Relations.— By plotting the measured stresses on the compressive and tensile faces of the column with respect to a fixed line, a graph of stress distribution across the section was obtained. After the proportional limit is passed on the compressive side this graph is no longer a straight line, and, finally, the part of the curve beyond the material yield point becomes asymptotic to the line representing the ultimate unit strength of the column. The gauge lines used in determining the stresses plotted are given on the graphs shown, so that by reference to Fig. 17, the section referred to will be readily identified. Fig. 21 contains examples of these curves for the data of Tables 8 and 9. It should be emphasized again that the curves presented are nearly identical with those of the other tests.

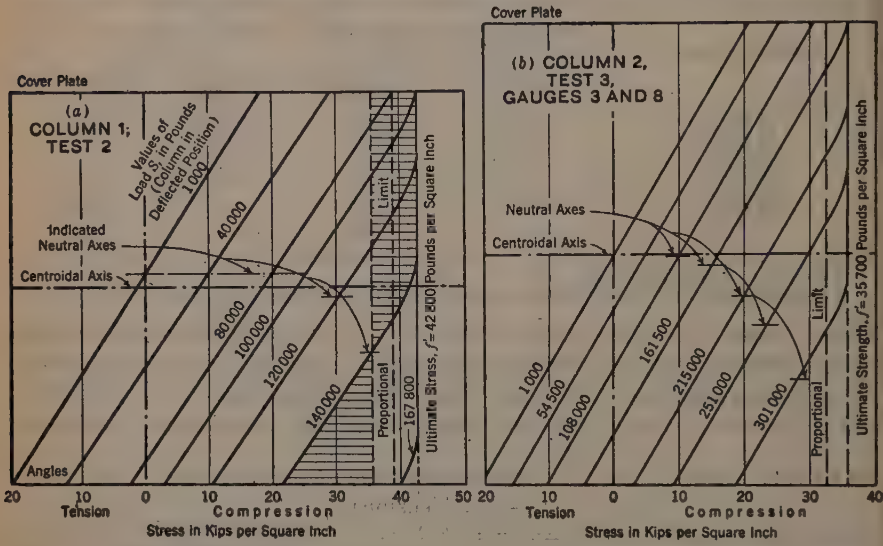


FIG. 21.—STRESS DISTRIBUTION (AVERAGE READINGS), COVER-PLATES AND BOTTOM ANGLES.

In Fig. 21(*a*), the curve for the 140 000-lb load has been shaded for the purpose of illustrating the re-adjustment more clearly. This curve shows a stress on the tensile face of 22 000 lb per sq in. in compression, and if the straight portion were extended, would give a stress of approximately 57 000 lb per sq. in. on the compressive face. Since the yield point was determined as 39 000 lb per sq in., and the ultimate testing strength as 42 800 lb per sq in., it is evident that such a high nominal stress cannot occur and that a re-adjustment consequent upon plastic flow must take place as indicated by curvature of the graph. The direction of the straight portion was made parallel, in all cases, to the last curve on which the end points were within the elastic limit. The shaded area represents the bending stress over the section, while the heavy dashed line (which intersects the curve at the indicated neutral axis) shows the axial stress. Fig. 21(*a*) shows this section to be almost completely under the action of axial stress at failing load.

Cover-Plate Stiffeners.—The views of Column No. 3, in Figs. 19(*e*) and 19(*f*) show a buckle occurring at the point of critical stress. (This column had a cover-plate relatively thinner than permitted by standard specifications.) The significance of this action will be discussed later. It will suffice here to note that it seemed worth while to investigate the feasibility of the application of a simple stiffening device to the plate. This was done in the case of Column No. 4 which (see Fig. 16) is a duplicate of Column No. 3, except for the plate stiffeners. No special attempt was made to develop the most practical device for commercial use; nor was any attempt made to follow a welding schedule and to reduce heating effects. The object in view was to test the effectiveness of the simplest type of stiffeners in supporting the wide, thin cover-plate when buckling was impending, so that it might suffer a certain amount of plastic deformation without "dishing" out of its original shape. Since little force is necessary to restrain such a plate from bending, small sections were welded to the cover-plate to give the necessary support against displacement normal to its plane, and thus, in effect, to cut down the free width. It was felt that if a simple device could be applied which would make the thin plate as resistant to buckling as a thicker one, this would be a desirable result regardless of the fact that in the specimens tested the plate buckling did not materially reduce the ultimate strength.

Discussion of Results

For specimens of the dimensions and character used in this study, it would be reasonable to expect that a high degree of integrity of section would be maintained under an intensity of stress corresponding to ordinary working values. On Column No. 1, however, a noticeable slip was observed between the web and the flange. The deformation difference amounted to approximately 30% of the bending strain. For the plate and channel sections used in Columns Nos. 2, 3, and 4, no direct measurement of the slip was made as in Column No. 1. However, since the stress-deflection values checked rather closely, it was concluded that no appreciable slip had occurred.

Fig. 19 shows strain lines occurring on both flanges of the specimen and across the web joining the two flanges. Since the nominal secondary stress due to bending was approximately 20 000 lb per sq in., such strain lines could not have occurred on the tensile side, under elastic conditions, until the average axial loading had reached 52 000 lb per sq in. (the yield point being approximately 32 000 lb per sq in.). As the ultimate axial load was approximately 35 000 lb per sq in., it is evident that the secondary bending was relieved by the plastic condition on the compressive face, which permitted a given angular change to take place with a much lower extreme fiber stress. This angular change, in turn, reduced the tensile flexural stress, with the result that the stresses due to the axial load of 35 000 lb per sq in. were sufficient to cause appreciable yield. That a complete re-adjustment resulting in a nearly uniform distribution over the section was the final state of stress, is evident from an examination of the strain lines on the tensile cover-plate, which, while nominally holding a stress of 15 000 lb per sq in., passed the elastic limit despite the fact that the ultimate average axial stress was only a little greater than the value of the elastic limit as measured.

The stress distribution curves (of which Fig. 21 is characteristic) bear out further the action indicated by Fig. 19. They show a redistribution of stress within the plastic range in complete agreement with the theoretical analysis presented previously. As the loading is increased, the neutral axis is progressively lowered until it lies near the edge of the section; the bending stresses become negligible and the distribution is nearly uniform over the entire section.

Referring to Fig. 2 and recalling that the plastic deformation in the neighborhood of the yield point is approximately fifteen times the elastic deformation at the proportional limit, it is obvious that it would require far greater secondary bending than the value developed in the test to cause the extreme fiber stress to exceed the yield point; that is, to reach Point *C*, Fig. 2. Assuming elastic behavior, the test loading gave a nominal maximum extreme fiber stress of 55 000 lb per sq in. The corresponding deformation, however, was less than twice the value occurring at the proportional limit; for the stress to reach the value, *C*, would require six to eight times this deformation.

Another point tc be emphasized is the fact that if the stress is assumed to remain at an approximately constant value in the compressive face while the yield is traversing the flat part of the curve of Fig. 2(*b*), even though the material flows freely without appreciable increase in stress, it can deform only to a limited extent because of the action of the remainder of the section; and, until such time as the entire section is approaching the yield point, it will deform only enough to get relief from additional stress, while continuing to hold its yield-point value. This condition is radically different from that of a specimen subjected to a stress that cannot be relieved in intensity by a slight yield.

In examining the distribution curves it is of interest to note that when yield on the compressive face is shown by a curvature of the graph, the increment of stress received by the tensile plate for uniform increments of loading is constantly increasing. A measurement of this change gives an idea of the "relief" of the bending stresses as plastic flow permits the column to assume the deflected position that originally caused the secondary stresses. A further estimate of this relief is obtained from Table 4, which gives the measured transverse force, *P*, necessary to cause the original deflection and its value during the test as plastic flow was reached. The reduction shown is quite marked. The amount of bending stress remaining (as shown by the stress diagrams of Fig. 21) may be checked by comparison between the permanent set in the column after test and the deflection used for secondary stress, the difference in deflection being used in an approximate calculation of the bending stresses necessary to cause this small deflection. The values of the permanent set are given in Table 10.

An examination of the data presented in Tables 8 and 9, and in the plotted graphs, shows a deformation along the center line of the column somewhat greater than that at the rivet lines. Since the bending stresses are transmitted to the plate at the rivet lines, such a distribution would appear, at first glance, to be contrary to what should be expected. As a possible partial explanation, it was noted that on all specimens a marked transverse curvature of the cover-plate (concave outward) developed under bending action alone, somewhat analagous to that commonly observed in wide flanged I-sections. This dishing inward was largely confined to the region of high flexural stress, and apparently resulted in an appreciable accentuation, locally, of the longitudinal curvature of the plate, which would be reflected in a somewhat higher deformation in the outside fibers along the center line of the column, where the gauge readings were taken.

TABLE 10.—PERMANENT SETS AND MAXIMUM LOADS

Column No.	Test	Deflection produced, in inches	Maximum load, *S*, in pounds (column in deflected position)	Permanent set, in inches
1..........	1	0.0900	140 000	0.0796
1..........	2	......	167 800	0.2033
2..........	3	0.2685	339 000	0.2055
2..........	4	0.2685	381 800	......
3..........	5	0.2556	271 000	0.1530
3..........	6	0.2556	344 000	0.2360
4..........	7	0.2556	295 500	0.1945
4..........	8	0.2556	305 000	0.2314

It is worthy of note that this dishing action has one distinctly favorable effect on the ultimate buckling strength of the cover in that, by directing the buckling inwardly, it reduces the unsupported width of the plate from the distance between the rivet lines to a value considerably less—probably almost the clear distance between the backs of angles or channels. It will also be observed that the bending of the column as a whole has a tendency to direct the buckling of the cover-plate inward. These facts are of considerable significance for cases in which local buckling is likely to be the criterion of ultimate strength.

In the general analysis the results of buckling formulas for ratios of thickness to unsupported width of one-fortieth and one-fiftieth were presented. Reference to Table 1 shows the $\frac{b}{t}$-ratios of the specimens tested to have varied from 32 to 48. The aforementioned results are quite applicable to these cases and clearly indicate that buckling is improbable in such cover-plates at stresses under the yield-point limit. The preceding discussion shows that stresses in execess of this limit are not realized under the most extreme conditions herein considered. In connection with this point it must be noted that the general buckling formulas become invalid as soon as the stresses pass the limit of proportionality, and furnish only roughly approximate values. For such cases the yield-point stress is usually considered the buckling stress. It is with regard to the latter point that the tests are significant. It might be reasoned that the wide thin cover is in an unstable condition when the stress reaches the yield point (Point B', Fig. 2(*b*)), and that local buckling of this cover should cause collapse of the entire section at a value slightly greater than the yield-point stress of the cover. However, in all cases except that of Column No. 3, despite the fact that a deformation twice that corresponding to the yield-point (Point B', Fig. 2(b)) stress was realized, there was no sign of buckling until the average stress over the entire section reached the yield point, and unlimited distortion became possible. In Column No. 3 where the $\frac{t}{b}$-ratio was $\frac{1}{48}$ (well below the limit permitted by most specifications), incipient buckling action was noticeable shortly after the extreme fiber had passed the proportional limit. However, the development of the buckle was so slow that the column as a whole maintained its integrity until, as in cases of other specimens, the average stress over the entire section attained a value slightly above the yield point.

It is believed that the reason for this action lies in the previously discussed fact that the distortion corresponding to entirely free plastic flow cannot take place in the cover, until the average stress is of sufficient intensity to cause the entire section to behave plastically. It is reasoned, therefore, that as soon as the cover reaches the yield point, and plastic flow takes place, the member assumes the deflected form with a greatly decreased resistance, the secondary stresses largely disappear, and the stress approaches uniform distribution across the section. Simultaneously, the distortion and direction of impending buckle of the cover-plate are controlled so as to permit the necessary deformations without actual buckling, thereby giving approximately the full yield-point stress as the ultimate load-carrying capacity of the section.

It has been noted that indications of buckling of the thin cover-plate in Column No. 3 were observed at an earlier stage of loading than for Columns Nos. 1 and 2, and that for an identical section in Column No. 4 simple stiffening devices were provided. The effectiveness of these stiffeners in holding the thin cover-plate in line in the region of high stress is clearly demonstrated. A comparison of Test 5, Column No. 3, with Test 7, Column No. 4 (the columns being identical except for the cover-plate stiffeners on the latter) shows that at the average unit stress causing a marked incipient buckling in Column No. 3, Column No. 4 showed no sign of such effects. While Test 8 (Column No. 4) showed a lower final value than Test 6 (Column No. 3), this was due to the fact that the former was not loaded to the point of complete collapse. At a load of 305 000 lb axial stress, however, the cover-plate was in better condition than that of Column No. 3 at the same load.

The results of the test indicated some serious defects in the particular stiffening device used. The sudden change in section at the end of the stiffener created a marked local field of stress concentration, the occurrence of which was indicated by the appearance of strain lines at a load well under the proportional limit of the column as a whole. It is clear that if such a stiffener is used, it should be extended farther toward the end of the column, or its section should be gradually tapered off to avoid such a marked change in cross-sectional area of the plate, since in the present case the unit stress in the plate adjoining the stiffener was practically equal to the maximum stress (stiffener and plate) at the center of the column. Further study and tests are required to determine the most effective form of stiffener, but the indications of this test are that a simple and inexpensive device of this type could readily be developed should it appear desirable to do so.

Summary and Conclusions

From the general analysis presented, the following significant features of secondary stress action may be summarized:

1.—Secondary stresses are limited and controlled by certain deflection quantities (see Equation (3)). For any particular loading condition, these deflections are (for primary unit stresses within the elastic limit) approximately proportional to the axial distortions of the truss members, considered as pin-connected, and when, for a given loading, these deflections have been attained, there is no further tendency for the stresses to increase.

2.—As long as both axial and bending stresses remain within the elastic limit and the transverse deflection is small, the behavior of a member under primary stress and secondary bending is closely analogous to a similar member under eccentric axial loading.

3.—If the maximum extreme fiber stresses pass the proportional limit, or if the member is of such type that the transverse deflection is large, the preceding analogy ceases to hold, since, in the case of secondary action, it is the deflection and not the moments that are proportional to the load. In particular, when the extreme fiber stresses reach the neighborhood of the yield point, a radical re-adjustment takes place, greatly relieving the flexural stresses.

4.—As a result of the re-adjustment in stress-strain relations, it may be inferred:

(*a*) That any tension member tends, as the load increases, to a condition of uniformity or stress over the cross-section, except for a local over-strain in a short section near each end.

(*b*) Compression members bent in single curvature may be seriously affected as regards $\frac{L}{r}$-failure if the transverse deflections due to secondary bending become large. This can only occur, however, in the case of large secondary moments and flexible members, a combination that is not ordinarily realizable. It was noted previously (see heading "Compression Member Bent in Single Curvature") that for values of $\frac{L}{r} \leqq 70$, the effect of secondary action in inducing $\frac{L}{r}$-failure is small — well under that corresponding to a pin-ended column with normal "equivalent eccentricities" at the ends. In such a case the rigid joint action which gives rise to secondary stress acts as a brake on the long column deflection before the point of ultimate column strength is reached.

(*c*) For a column bent in double curvature, the secondary action, by forcing the curvature into two "waves", may actually have a beneficial effect as regards $\frac{L}{r}$-failure.

(*d*) For "stocky" columns $\left(\text{say, } \frac{L}{r} \overline{\overline{<}}\ 40\right)$, which are ordinarily the only members that develop high secondary stresses, failure is nearly always due to local over-strain. Until the average stress approaches the yield point, the transverse deflection is negligible, and column action in the ordinary sense cannot occur. For such columns the secondary stresses, whether resulting in single or double curvature (almost invariably the latter will be the case for high secondary stresses) will merely result in high stresses on the compressive face, which are rapidly relieved by plastic flow of the material as the yield point is approached.

5.—It might be expected that a combination of primary and secondary stresses beyond the proportional limit would adversely affect the stability against local buckling of the wide, thin cover-plates commonly used in the compression chords of bridges. An analysis of the problem indicates, however, that local buckling is unlikely to occur, in plates having a $\frac{t}{b}$-ratio consistent with the requirements of standard specifications, until the average stress over the column approaches the yield point of the material, which, in all compression members, marks the ultimate strength of the member.

The tests made primarily for the purpose of exhibiting the behavior of the box type of compression member with respect to local failure under primary stress and a high percentage of secondary stress, have been discussed rather fully in the previous pages. The test results may be summarized briefly, as follows:

6.—Re-distribution of stress beyond the proportional limit was found to take place in a manner closely agreeing with theoretical analysis.

7.—For combinations producing nominal values of extreme fiber stress as great as 55 000 lb per sq in. (f_p = 35 000 and f_s = 20 000 lb per sq in.), it was found: (*a*) That while strain lines began to appear in the cover-plates as the proportional limit was passed, no indications of local buckling appeared until the average stress over the section approached the yield point of the material, for plates with $\frac{t}{b} = \frac{1}{32}$ and $\frac{1}{42}$; but, (*b*) for plates with $\frac{t}{b} = \frac{1}{48}$, some evidence of incipient local wrinkling was noticed at a load somewhat less than the ultimate capacity of the column, although this had apparently no appreciable effect on that value.

8.—When stiffeners were applied to the cover-plates of Column No. 4 (see heading, "Cover-Plate Stiffeners"), the indications of incipient buckling noted in Column No. 3 (Summary Item 7(*b*)) were entirely absent.

Although the general analysis of the behavior of bridge members under combined primary and secondary action resulting in local stresses beyond the proportional limit is believed to apply without restriction, it is obviously impossible to draw valid general conclusions from the test results on four specimens. A much more comprehensive program of experimentation would have to be undertaken before a final pronouncement could be made. Subject to these limitations, the following significant conclusions are indicated:

(*a*) The ultimate practically utilizable strength of a tension member is not affected by any secondary stress action within reasonable practical limits.

(*b*) Compression members sufficiently flexible to develop $\frac{L}{r}$-failure before the yield point is reached, will usually exhibit too low a value for the secondary bending to reduce the ultimate carrying capacity of the member materially.

(*c*) The rigid type of built column, in which alone high secondary stresses are to be expected, almost invariably fails by local over-stress, usually at a point at which local buckling can readily take place. High secondary stresses have no effect in hastening such local failure, and, therefore, have no effect in reducing the actual ultimate strength of such compression members, providing the section proportions are governed by the limits of present standard specifications.

The results of the investigation do not justify the conclusion that secondary stresses are of no importance. The fundamental principle which has so largely governed structural design (that no fiber stresses should exceed the yield point of the material), is too firmly established and is supported by too many sound reasons to be lightly ignored. Such a principle, however, cannot be applied blindly; it is of the utmost importance for the designer to know the consequence of overstressing any part of the structure. This may mean general failure, a limited, localized failure not endangering the structure as a whole, or merely undesirable local distortion and permanent set. The practical significance of this investigation lies in the fact that it indicates that overstressing due to secondary bending falls in the latter class. This is of vital importance in fixing the unit stresses and otherwise determining the margin of safety for cases in which high secondary stresses are involved.

The investigation does not cover certain important by-products of secondary bending as, for example, the effect on riveted and welded connections of alternating secondary moments. For certain types of joints, this effect may be quite important. Due to this and perhaps other effects, and the general desirability of avoiding large permanent set, it is highly desirable that the designer should know if and where high secondary stresses occur in a structure. Under certain conditions, he may deem it desirable to modify the design to reduce such stresses or to provide for them specially in the detail design. In any case, it is a matter of great practical designing significance to be assured that an unforeseen and unexpected overload, producing a combination of primary and secondary stress nominally far above the yield point of the material, will not endanger the safety of the member as long as the average stress over the section is maintained safely below the yield limit.

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PAPERS

THE SPRINGWELLS FILTRATION PLANT, DETROIT, MICHIGAN

BY EUGENE A. HARDIN[1], M. AM. SOC. C. E.

SYNOPSIS

The design and construction of a filtration plant and filtered water reservoir at Springwells Station is described in this paper. These units form a part of the additional works constructed during 1929-1932 for the Board of Water Supply of the Metropolitan Area of Detroit, Mich. The filtration plant is of the rapid sand, gravity type, having an ultimate capacity of 300 000 000 gal daily. The reservoir is in two sections each of which has a capacity of 20 000 000 gal, with provision for a third section to be added later. The general basis of design, the data pertaining to the various parts of the plant, the hydraulics of plant flow, the construction program, and the tabulation of both construction costs and engineering costs are outlined herein.

The water supply for the City of Detroit is the largest to be completely filtered by rapid sand filtration, and includes the two largest rapid sand filtration plants in the world—the Water-Works Park Filtration Plant, having a maximum daily capacity of 360 000 000 gal, and the Springwells Filtration Plant, having a maximum daily capacity of 300 000 000 gal (capacities based on a filtration rate of 180 000 000 gal daily per acre).

HISTORY OF WATER TREATMENT AT DETROIT, MICHIGAN

The City of Detroit has taken its water supply from the Detroit River for more than 100 years. Although privately owned and operated for the first eight years of their existence, the water system and supply works have been owned by the municipality since 1836. Since that time, from a small city of 6 900 inhabitants, the district served has increased in population to more than 2 000 000 and in area to more than 100 sq miles, extending about 10 miles along the river and about 10 miles inland from the river. Improvements for

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NOTE.—Discussion on this paper will be closed in February, 1935, *Proceedings*.

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increasing the quantity of the supply, and its distribution, were the main considerations, except in the location of intakes, until, in 1912, when purification of the supply was first considered and permanent provision was made for chlorination. The quality of the water in the Detroit River at the intake built in 1905 (named 1905 Intake) has been comparatively good, but with increasing pollution and the growing consciousness of the consumers of the value and need of improvement in the quality of the water, the desire for a purer and more palatable supply increased until, in 1917, an experimental filtration plant with a capacity of 200 000 gal per day was constructed and operated for more than a year, establishing the feasibility of filtering and developing popular approval by dispensing filtered water to those who came for it. One year of operation (from October 1, 1917, to September 30, 1918) was used as the basis of a report by the late R. Winthrop Pratt, M. Am. Soc. C. E., dated March 1, 1919, outlining the general design of a filtration plant, to treat the entire then existing supply. In 1921, contracts were let for filtration works at Water-Works Park Station (see Fig. 1) and the filtration plant was completed and put in service on December 22, 1923.

While it improved the quality of the water greatly, the filtration plant at Water-Works Park did not add to the quantity. Due to the rapid increase in size of the city in both population and area, it was evident that a far-sighted and comprehensive plan of additional supply for the Detroit Metropolitan Area was necessary immediately. Accordingly, a Commission of Consulting Engineers, composed of Messrs. George H. Fenkell, W. C. Hoad, Clarence W. Hubbell, Theodore A. Leisen, and the late Gardner S. Williams, Members, Am. Soc. C. E., was appointed, and reported jointly on January 5, 1924. This Commission recommended an extensive program, involving the construction of additional supply works, beginning immediately with (*a*) the building of a new intake in the river; (*b*) a raw-water tunnel from intakes to the west side of the city; (*c*) a complete new supply station at the site selected on the west side; (*d*) additional distribution mains; and (*e*) future construction of a third complete supply station at a site to be selected in the northeastern part of the city. Item (*c*) included recommendations for a low-lift pumping plant, a filtration plant, a filtered water reservoir, and a distribution pumping plant.

GENERAL DESCRIPTION OF FLOW

The new system provides for the distribution of additional water to the Metropolitan Area of Detroit from three separate stations, each having a filtration plant and the necessary pumping plants (see Fig. 1): (1) The old Water-Works Park Station on East Jefferson Avenue; (2) the Springwells Station on West Warren Avenue; and (3) a future Northeast Station (see Fig. 1). The water is taken from a new, lagoon type, intake built in the Detroit River just northeast of the up-stream end of Belle Isle and not far from the 1905 Intake crib, which will remain in service as an auxiliary intake, and will be carried through a system of gravity tunnels to the old station and across the city to the new stations. The intake, the Springwells Station, and the raw-water tunnel connections to Water-Works Park Station and Springwells Sta-

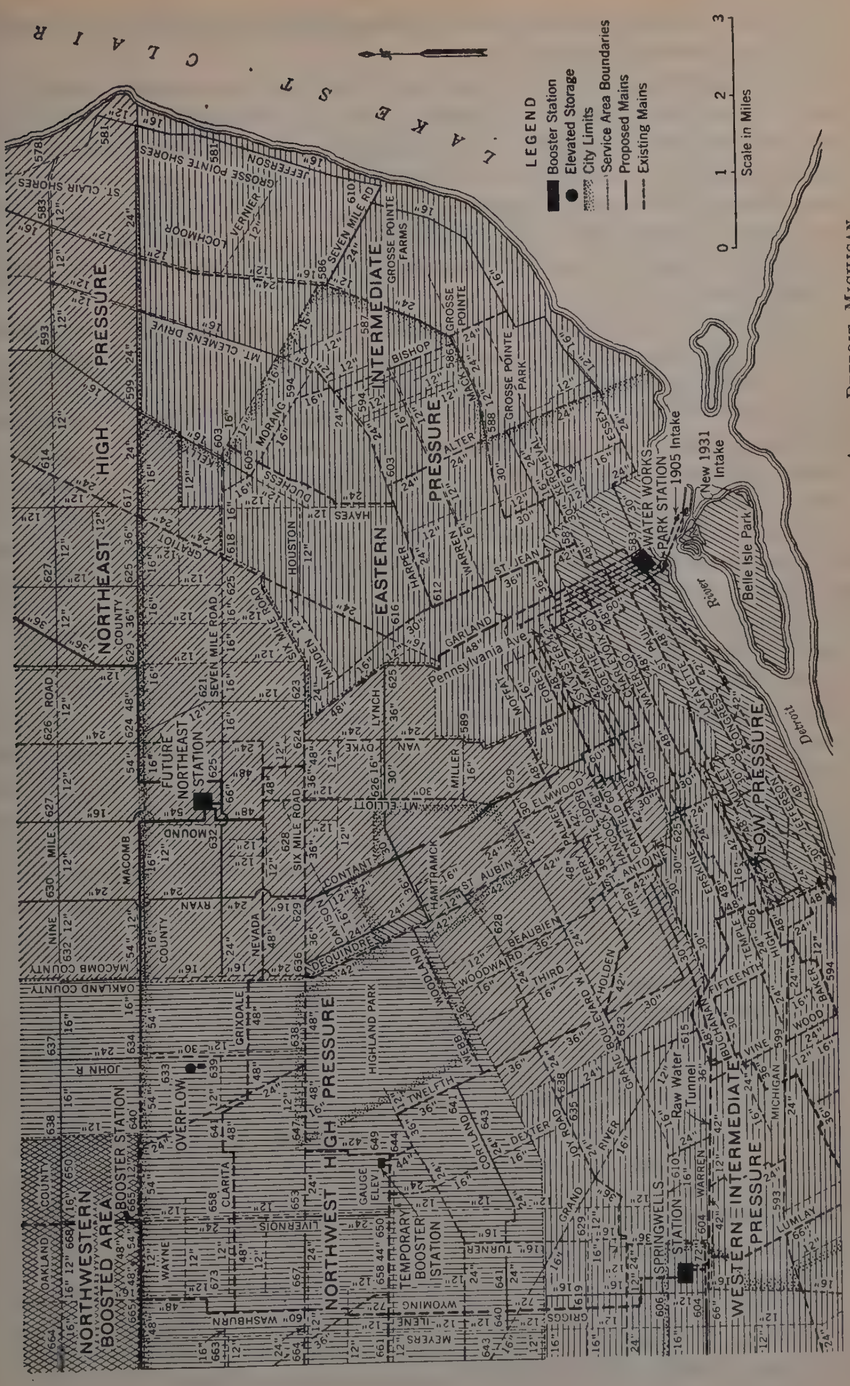

FIG. 1.—LOCATION OF PRINCIPAL WATER SUPPLY WORKS, METROPOLITAN AREA, DETROIT, MICHIGAN

tion are part of the present project. Northeast Station and the branch tunnel to it will be built at a future date.

The water to Springwells Station flows from the intake through 3 568 ft of concrete-lined tunnel, 15.5 ft in diameter, built in rock under the river, to a riser well and screen chamber on the shore at Water-Works Park; thence through 10 633 ft of concrete tunnel, 14.0 ft in diameter, in clay, north to the intersection of Pennsylvania and Forest Avenues; thence west through 44 705 ft of concrete tunnel, 12.0 ft in diameter, in clay, to the Springwells Station site, where it turns and enters the suction well of the raw-water (low-lift) pumping plant.

At this point the pumps elevate the raw water into the mixing chamber of the filtration plant through two 10 by 8-ft (or equivalent section) concrete conduits running behind the pumping plant and under the generator plant floor. The flow through the filtration plant is shown in Fig. 2. (The superstructures of the plant are not shown.) Just before entering the mixing chamber the water is metered through Venturi tubes cast in the concrete conduits. Coagulants or chlorine may be applied to the raw water at the entrance to the low-lift plant, at the entrance or exit of the raw-water meters, or at the entrance to the mixing chamber. Thorough and rapid dispersion of the chemicals through the water is accomplished by the turbulence in the pumps, meters, or entrance gates, as the case may be. In the mixing chamber the water is stirred by mechanical agitators and brick baffles.

The mixing chamber discharges into a conduit running the full width of the four settling basins, from which conduit the water may rise at two junction and gate wells which admit the water into the distributing channels of each basin. Entrance to the basins is through vertical slots spaced to give uniform distribution. After passing through these basins at settling velocities the clarified water is decanted from the surface by weirs, and flows into the filter building at two points through main conduits. At the filter operating galleries these conduits branch into the filter influent mains from which the inlets are connected to the sixty-eight rapid sand-filter units. The water flows on to the filter sand beds from the inlets and passes through 20 in. of sand and 18 in. of gravel to the perforated-pipe grid collector system below.

The filtered water that is collected is discharged through automatic rate-of-flow controllers into the main filtered-water conduits in each of the two pipe galleries between the four rows of filters. The two filtered-water conduits discharge into a weir chamber at their eastern ends where the water level is controlled to maintain submergence of the filter-effluent piping as well as the high suction level on the distribution pumps when operated according to the shunt system, described subsequently. From the weir chamber the water may flow either directly to the pumping plant or into the filtered-water reservoir, as flow conditions demand.

Circulation in the filtered-water reservoir is induced by a center baffle-wall in each section and inlets and outlets controlled by flap check-gates. Water is drawn from the reservoir through conduits entering the opposite end of the pumping plant to that connected directly to the weir chamber. The dis-

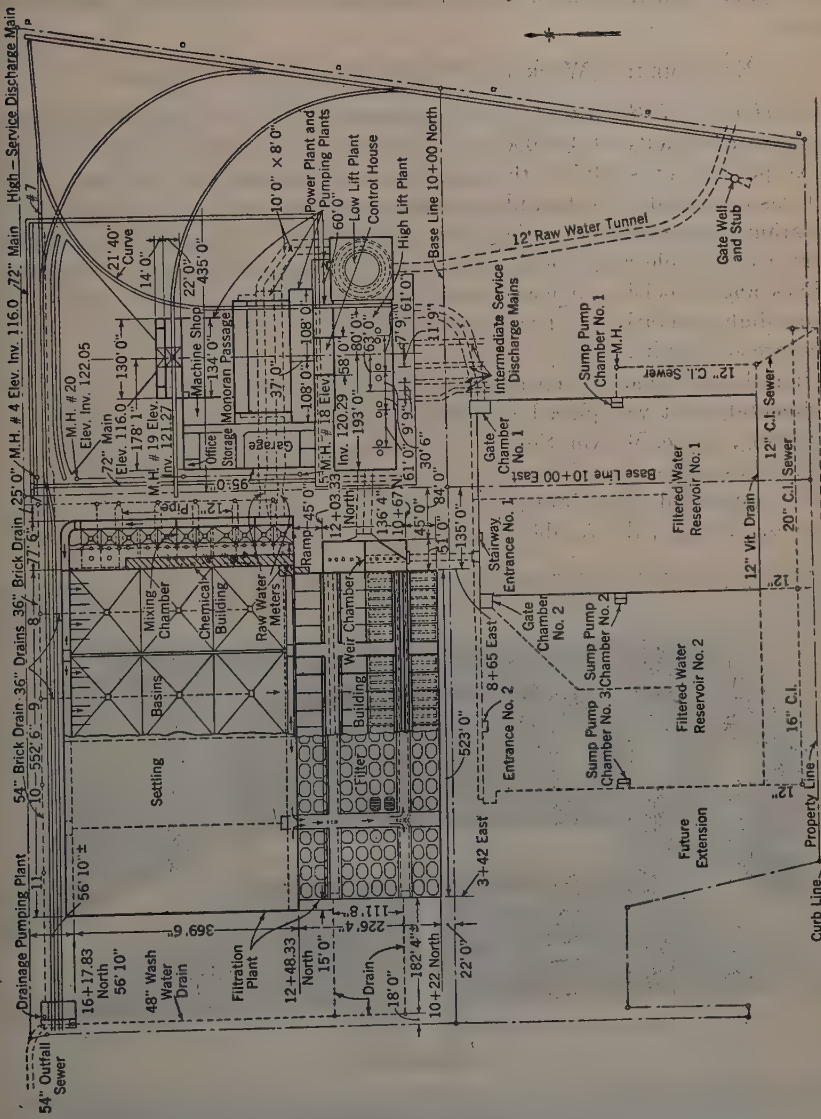

FIG. 2.—GENERAL PLAN, FILTRATION PLANT, SPRINGWELLS STATION

tribution pumps discharge through steel mains connected to each of two service districts (a high-pressure area and an intermediate pressure area, see Fig. 1), which are served jointly by the Water-Works Park Station and the Springwells Station. The detailed description which follows covers only the design and construction of the filtration plant and reservoir at Springwells Station.

Experimental Work and Studies Preliminary to Design

Experimental Filter Plant.—In the summer of 1925 an experimental filter plant with a capacity of 150 000 gal per day was designed and built for the express purpose of studying the phenomena of mixing and settling. This plant was composed of rectangular wooden tanks supported on a concrete mat foundation and enclosed in a light frame structure. The plant was arranged so that a reasonable range of times and velocities of mixing and a wide range of times and velocities in coagulation basins could easily be obtainable by making slight alterations. Two identical filter units were provided for use as a measure of results.

After about six months' operation in parallel with the large filter plant, the experimental plant was placed in charge of Mr. Roberts Hulbert and Frank W. Herring, Assoc. M. Am. Soc. C. E., who began on the program of settling-basin studies and obtained some very good results; but certain peculiarities in these results led to an intensive search to determine the reason for inconsistencies. Finally, the trouble was discovered in the fact that the two filter

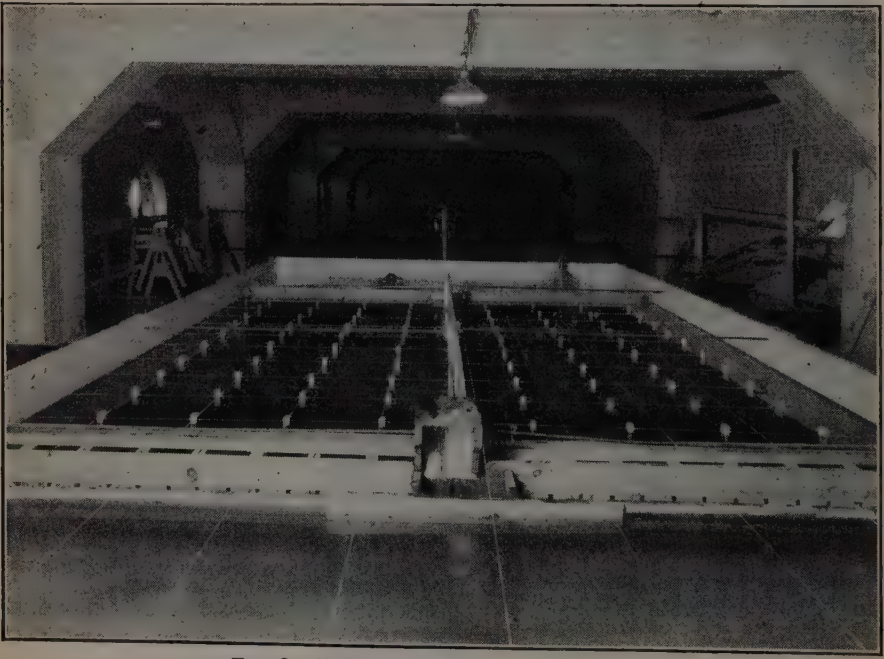

Fig. 3.—View of Settling Basin Model

units used to measure the results were not washed uniformly. Having become absorbed in the development of better methods of washing filters, the remainder of the time available for the experimental filter plant operation was spent on this subject and a technique was developed of washing them to a certain definite sand expansion, instead of at the usual constant rates of wash-water flow.[2]

Scale Model Tests of Settling Basin Inlets.—Concurrently with the operation of the experimental filter plant, investigations of flow in settling basins were made by the use of a 1:25 scale model replica of the basin proposed to be used at the Springwells Plant. This model (see Fig. 3) was the means of determining the detail of inlet and outlet construction required to give a uniform distribution of flow across the full width of the basin and the longest actual retention period, as determined by floats, dyes, and salt solutions. Potassium permanganate dye was most satisfactory in determining the condition and distribution of flow by observing the advancing cloud of colored water. The time element was denoted by plotting the front edge of the color cloud at the end of each minute after the application of the color until it flowed out at the outlet. This plotting gave a record of each run, such as the typical form shown in Fig. 4.

The outlet conditions were early found to have little effect on the flow distribution. Therefore, attention was concentrated on the inlet detail. From the test results on the model, a vertically slotted wall inlet was adopted, with training walls between pairs of slots, The arrangement of the entrance baffles is shown in Fig. 4, with seventeen slots of the size and spacing indicated. In the entrance chamber the openings are beveled on the down-stream side (see Fig. 4(*b*) and Fig. 4(*c*)). Guide-vanes were of $\frac{7}{16}$-in. galvanized iron. The basins have "straight-through" flow.

Filter Under-Drain Lateral Experiments.—To determine how uniform the distribution of wash-water flow would be from a tentatively adopted design of filter under-drain lateral and to develop a low-cost brass, or bronze, bushing for the perforations in the under-drain laterals, a testing apparatus was built adequate for one typical full-sized lateral, almost identical in dimensions with those that would go into the actual filters. Several types of pipe materials in this apparatus, were tested, as well as types of perforations and bushings. Incidental to this test, a series of submerged orifices was calibrated and discharge coefficients were obtained.

Analytical Investigations and Studies.—Analytical studies were made to determine the economic size of filter units, economic filtering head, the design of the wash-water supply system (with particular regard to the relationship of the size of wash-water tanks and wash-water pumps supplying those tanks), and the costs of types of filter bottoms, wash-water trough materials, and chlorine-handling equipment.

General Basis of Design.—The ultimate capacity of Springwells Station was determined by a detailed study of the distribution system and the proper division of load among the three plants designated by the consultant's report of 1924.[3] This report and the results of experimental work and preliminary studies, together with previous experience, led to adoption of values given in Table 1, as the basis of design for the Springwells Filtration Plant. The design required, furthermore, that coagulants should be applied to the initial mix at the pumps or the meters and that the mixing be by mechanical agitators. The design of the details of the settling basins was determined by scale model studies (see Fig. 4). The filter bottom specified was the perforated pipe grid type with a center manifold.

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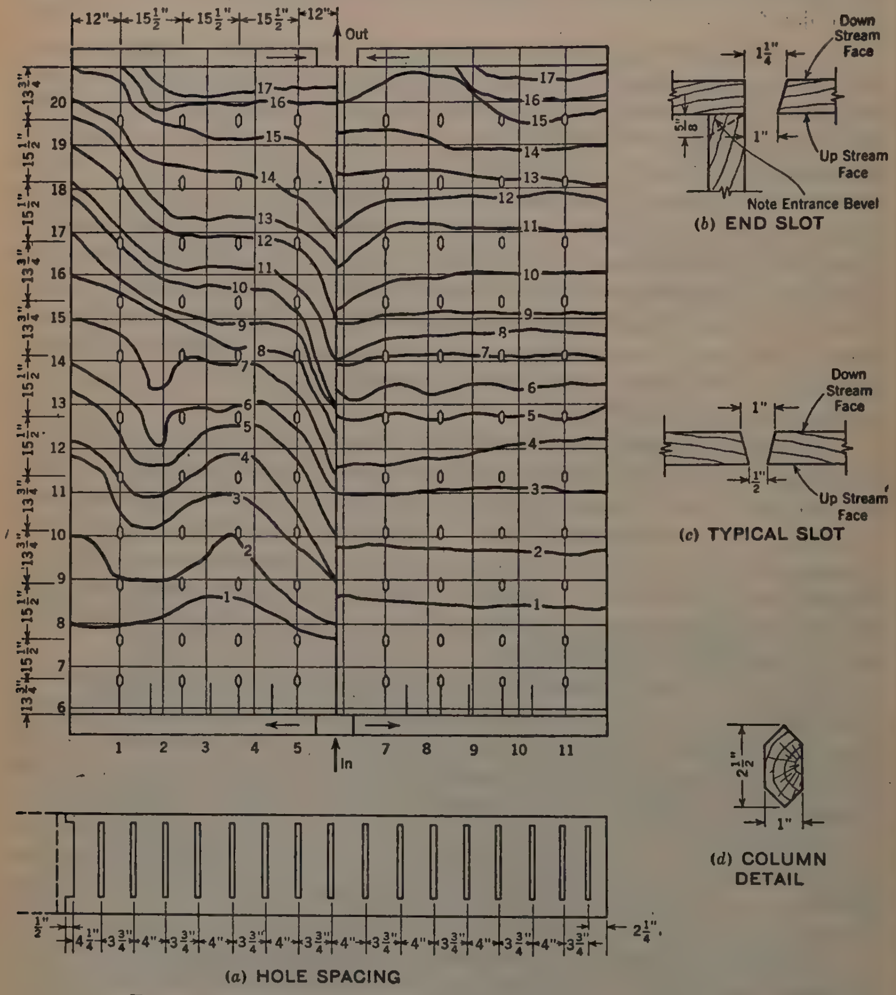
[2] "Studies on the Washing of Rapid Filters," by R. Hulbert and F. W. Herring, *Journal*, Am. Water Works Assoc., Vol. 21, November, 1929, p. 1445.



(*b*) END SLOT

(*c*) TYPICAL SLOT

(*d*) COLUMN DETAIL

(*a*) HOLE SPACING

FIG. 4.—DETAILS OF MODEL AND TYPICAL RECORD OF TEST RUN

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[3] "Trunk Main Water Distribution System for Greater Detroit," by L. E. Ayres, M. Am. Soc. C. E., *Engineering News-Record*, Vol. 100, No. 22, May 31, 1928, p. 855.

TABLE 1.—DESIGN FACTORS: SPRINGWELLS FILTRATION PLANT

Item No.	Description	Value
	(a) ULTIMATE CAPACITY (ESTIMATED AS OF 1955)	
	Net Output, in Million Gallons Daily:	
1	(a) Average day	206
2	(b) Maximum day	278
3	Six filters supplying wash water, in million gallons daily	27
4	Required gross capacity, in million gallons daily	305
	Filtered-Water Reservoir, in Million Gallons:	
5	(a) Ultimate	60
6	(b) Present	40
	(b) MIXING AND COAGULATION	
7	Time of mixing contact at maximum capacity, in minutes	15
8	Number of mixing chamber units	3
	(c) SETTLING BASINS	
9	Time of retention at maximum capacity (300 mgd), in hours	2
10	Velocity in basins, in feet per minute	2 to 3
	Maximum Velocity, in Feet per Second:	
11	(a) Inlets	1
12	(b) Outlets	1
13	Number of basin units	4
	(d) FILTERS	
	Rate of Filtration, in Million Gallons Daily per Acre:	
14	(a) Normal	160
15	(b) Maximum	180
	Rate of Washing, in Inches of Rise per Minute:	
16	(a) Normal	36
17	(b) Maximum	39
	(d) FILTERS (*Continued*)	
	Size of Each Unit:	
18	(1) Area of sand, in acres	$\frac{1}{40}$
	(2) Filtering rate, in million gallons daily:	
19	(a) Normal	4.0
20	(b) Maximum	4.5
21	Number of filter units	68
22	Maximum velocity of influent, in feet per second	1
23	Operating head, in feet	9.5
24	Capacity of wash-water tank, in million gallons	0.1
25	Capacity of wash-water pumps, in gallons per minute	28 000
	(e) STRUCTURAL LOADS AND STRESSES	
	Weight, in Pounds per Cubic Foot:	
26	Water	62.5
27	Earth	100.0
28	Concrete	150.0
	Maximum Allowable Unit Stress, in Pounds per Square Inch:	
	(1) Tension in reinforcement, for:	
29	(a) Floors, walls, and footings	16 000
30	(b) Roofs and Columns	18 000
	(2) Compression in concrete, for:	
31	(a) Floors, walls, and footings	650
32	(b) Roofs and tied columns	750
33	(c) Circular columns with spirals	1 000
34	Maximum allowable load on a bearing pile, in tons	25
	Maximum Allowable Bearing Value of Earth Foundations, in Tons per Square Foot:	
35	(a) Filtration plant structures	0
36	(b) Reservoir	1

Horizontal earth pressures were assumed to be equivalent to hydrostatic pressures, plus surcharge when imposed. Live loads were determined by conditions in substructures; in superstructures, live loads were determined by conditions, or by reference to the Detroit Building Code, whichever was the higher.

DESCRIPTION OF PLANT

General.—The keynote of the entire design is practical simplicity. Attention was given to each detail to avoid complex situations which would be costly to build, or which later might cause trouble in operation and maintenance. Where auxiliaries have been used, such as master control for filters, summation devices for meters, float-controlled water-level indicators, float controls for pumps, etc., they have been installed so that their discontinuance from service will not interrupt the operation of the plant or affect the functioning of the standard equipment with which they are used. As a rule, automatic devices, such as the differential chemical feed, the automatic filter shut-off, the proportional chlorine feed, etc., have been avoided, preference being given to manual control where practicable.

The construction of the conduits, chambers, basins, filters, and reservoirs, is of reinforced concrete. The superstructures have structural steel frames encased in concrete (with the exception of the steel frame of the Chemical Building which is only partly encased), reinforced concrete roofs insulated with cork and tar and gravel roofing, and hollow-tile curtain-walls faced on the outside with Indiana limestone and on the inside with terra cotta, glazed brick tile, or glazed hollow tile. The operating floors of the filters are finished with 6 by 6-in., red, quarry tile with dark green terrazzo borders. The floors in the Chemical Building, storage rooms, etc., are finished with concrete hardened with granite chips and ground smooth.

The entire structure is supported on a foundation of wooden bearing piles driven to hardpan through a subsoil of plastic and soft blue clay. The total load averages 20 tons per pile with the piles arranged so that, when the plant is entirely filled and loaded, there is a variation of not more than 5 tons per pile. On account of the unfavorable subsoil conditions, care was taken to avoid excessive concentrations of loads in the plant. Some variation in loading is occasioned by fluctuating live loads, such as in storage bins, basins, filters, and wash-water tanks.

Chemical Plant.—The Chemical Building is over a part of the mixing chambers, and contains the chemical storage tanks, chemical unloading and conveying equipment, the dry-feed machines, and hoppers for dosing ground sulfate of aluminum and ammonium sulfate. The arrangement of chemical equipment is shown in Fig. 5. This equipment is arranged so that, by a minimum of revision, the manufacture of aluminum sulfate may be provided for, at the same time preserving the dry alum equipment for stand-by service. The provision for alum manufacture contemplates making the alum syrup and feeding it as a solution.

The design of the Chemical Plant was based on the following capacities:

(1) At maximum demand, a storage capacity of 30 days' supply of raw materials.
(2) At normal demand, a storage capacity in each alum feeder bin, of 1 day.
(3) At maximum demand, a storage capacity in solution feed tanks, of 1 day.
(4) At maximum demand, a storage capacity for alum syrup, of 3 days.
(5) Unloading equipment to handle a 40-ton car in 8 hr.
(6) Reclaiming conveyors sufficient to handle 1 day's supply in 3 hr.

The equipment installed in the Chemical Plant consisted of the following:

(1) Suspended steel storage tanks of 390-tons total capacity consisting of:

  (*a*) Four storage tanks at 60 tons capacity each.
  (*b*) Three storage tanks at 30 tons capacity each.
  (*c*) Five feeder hoppers at 12 tons capacity each.

(2) Pneumatic conveyor for unloading from railroad cars and delivering to storage (capacity 5 tons per hr).

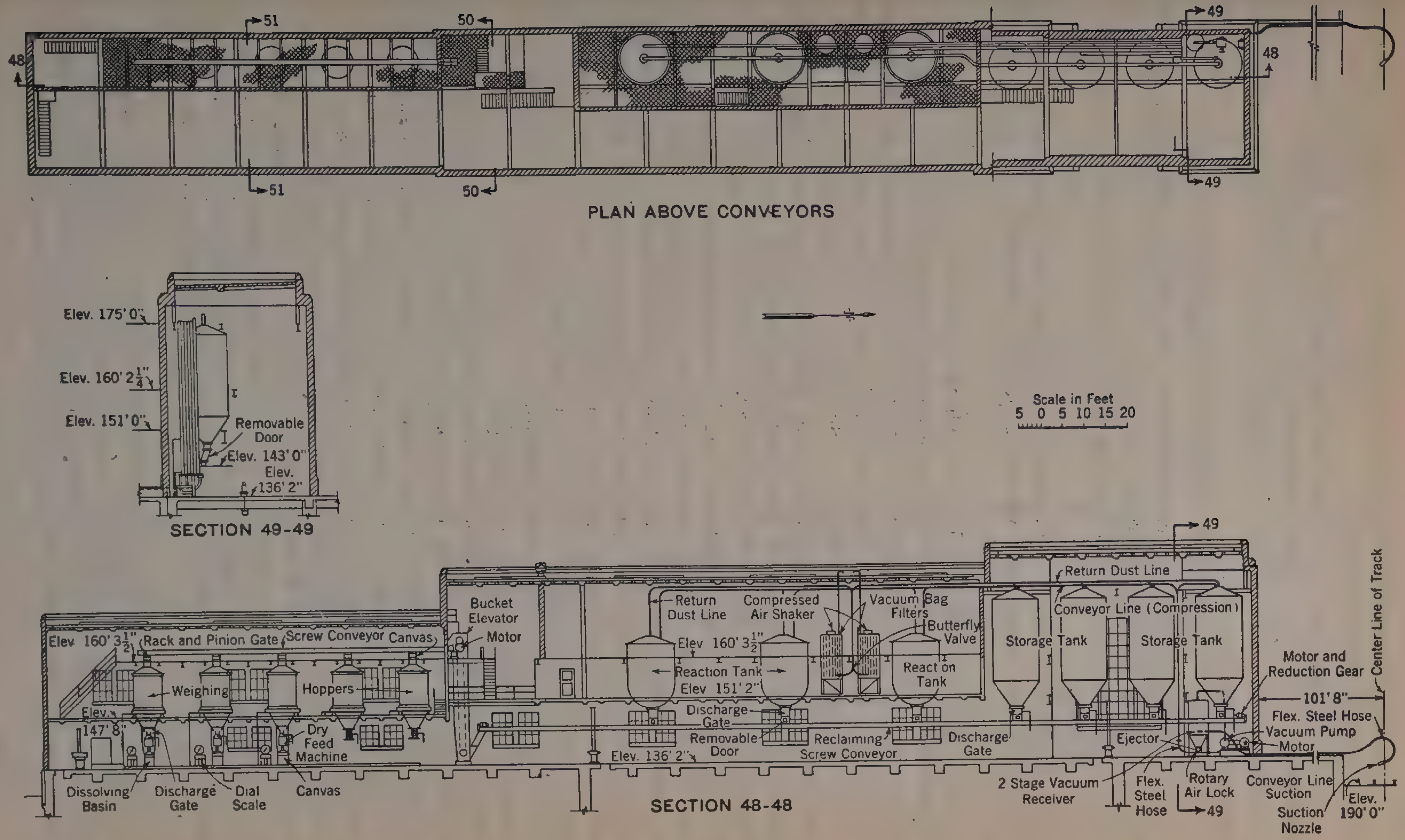

Fig. 5.—Arrangement of Chemical Equipment

(3) Screw conveyor, 200 ft long, for reclaiming material from storage tanks (capacity 5 tons per hr).

(4) Bucket elevator for lifting material from the reclaiming conveyor to the conveyor above the feeder hoppers (capacity 6 tons per hr).

(5) Screw conveyor for distributing material to the feeder hoppers (capacity 6.6 tons per hr).

(6) Five dry-feed machines suspended from the feeder hoppers.

(7) A weighing scale for each feeder hopper and dry feed machine, with 30-in. indicator dial and recorder.

(8) A dissolving basin for each dry-feed machine, delivering to the chemical dosing lines.

(9) Three 3-in. dosing line headers of 99% pure copper extra heavy tubing, with stream-line fittings and valves of bronze composition resistant to the action of the alum solution.

Raw-Water Conduits and Meters.—The raw water is delivered to the mixing chamber through two 10 by 8-ft concrete conduits which run under the floor of the generator room in the power plant. At the west side of the power plant the conduits change shape to 9-ft square conduits and slope upward on a grade of about 12 per cent. In this section, a distance of about 60 ft, a raw-water meter is installed in each conduit. These meters are of the Venturi type with 9-ft square inlet and outlet ends and 4-ft square throats. The meters are cast of concrete with bronze-lined iron throat and pressure-ring castings and bronze-lined cast-iron inlet pressure rings set in the concrete. The raw-water meter registers are in a passageway of the building convenient to both alum and chlorine feeders. Dosing points for coagulating chemicals and chlorine are provided at both inlet and outlet ends of the meter tubes. In the design of the meter tubes advantage was taken of the upward slope in the conduits so as to compensate for their convergence in such a way that no air is trapped, on filling the conduits, and all water may be drained out, on emptying the conduits.

Mixing Chambers.—The dosed raw water enters the mixing chambers from the down-stream ends of the two parallel raw-water meters. The mixing chambers are in three units of approximately 5-min retention period, each with a by-passing channel. The general arrangement and details are shown in Fig. 6. The water enters the by-passing channel on the east side which is equipped with gates to shut off or admit water to any of the three chambers. Normally, the water enters through the three 6 by 10-ft sluice-gate inlets to the south chamber, flows northward through the three chambers (the dividing wall-gates of which are normally open) and leaves through the three 6 by 10-ft gates at the northwest corner into the coagulated water conduit to the settling basins.

All the sluice-gates in the mixing chambers are of the rising stem type, and are hand-operated, with worm-geared floor stands. While it is in the mixing chambers the water is stirred by mechanically driven paddles rotated at a peripheral speed of 0.67, 1.0, 1.33, or 2 ft per sec, the speed being regulated

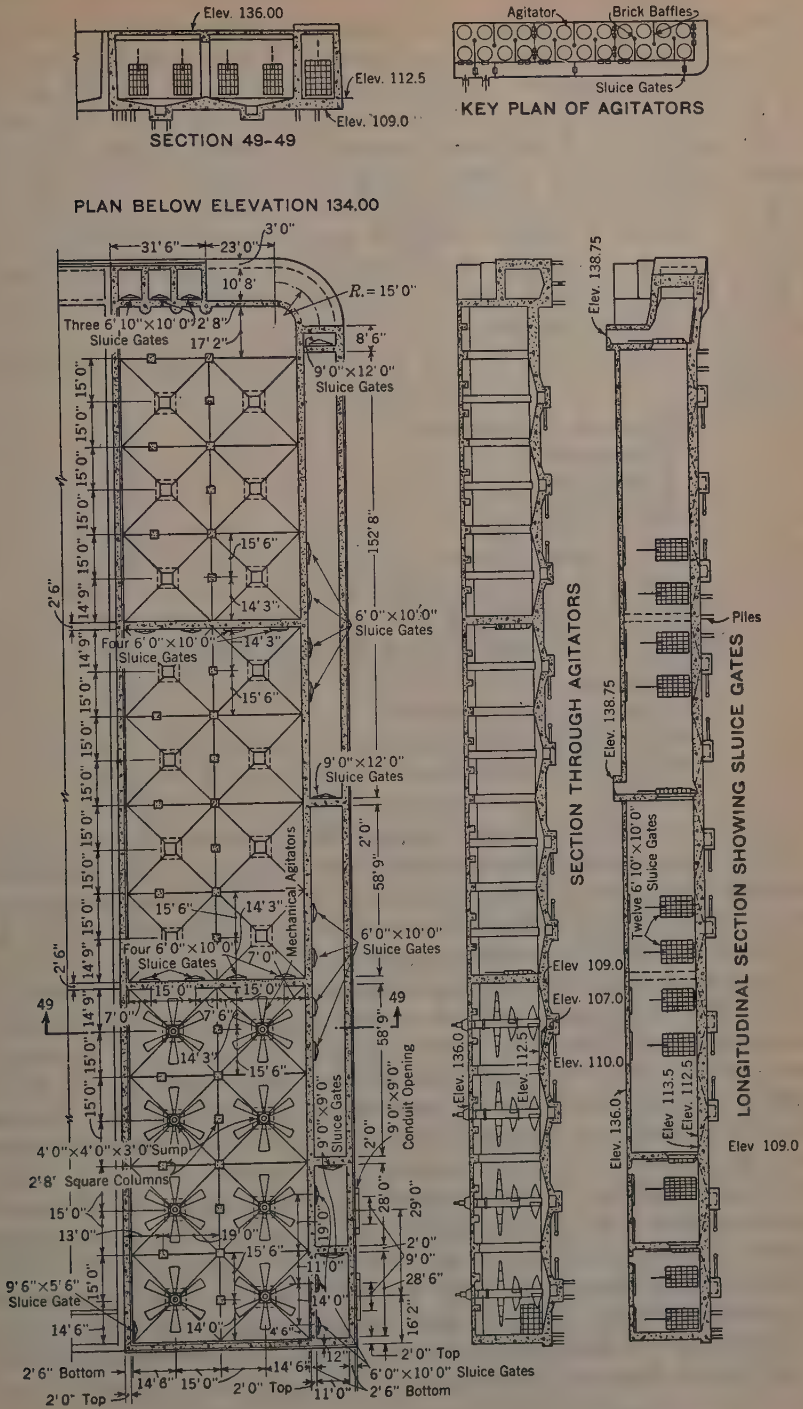

FIG. 6.—PLAN AND DETAILS OF MIXING CHAMBERS

to give the best conditioning of the water. At present (1934), only the first mixing chamber is equipped with agitators, and brick baffle walls have been built in all three chambers. As the plant load is increased the other chambers will be equipped with agitators. The location of the mechanical agitator units for the south section of the mixing chamber is indicated in Fig. 6. Steel-plate paddles, 10 ft long, are mounted spirally about a shaft of 12-in. steel pipe hung from a thrust-bearing in a base casting mounted on the cover slab of the chamber. The lower end of the shaft is steadied in a sleeve-bearing attached to the bottom of the chamber sump. This paddle assembly is driven through a 900:1 vertical reduction gear by a four-speed squirrel-cage vertical motor mounted on the gear housing.

The following data will serve as a summarized description of the mixing chamber:

Elevations, in Feet, Above:	
High water	131.6
Normal water	131.1
Bottom Slab:	
At wall line	112.5
At edge of sumps	110.0
Volume of water, in million gallons	3.201
Average dose of alum, in grains per gallon	0.6
Time of mixing, in minutes, at 300 mgd capacity	15
Length of horizontal travel, with baffles, in feet (approximately)	1 000
Velocities, in Feet per Second:	
Of horizontal flow, at 300 mgd capacity	1.11
Of agitators (peripheral)	0.67 to 2.0
In gates	2.5 to 4.0

Settling Basins.—The general features of the settling basins are shown in Fig. 7. Referring to Fig. 8, column vanes are 12 in. thick at Elevation 112.5 and 6 in. thick at the top, from Point *A* to the entrance baffle. Intermediate vanes are 6 in. thick at the top and 12 in. at Elevation 112.5 for their full length and the batter continues to the top of the vane.

The settling basins are in four units each about 135 by 340 ft in plan, with about 18 ft of water depth and 5 ft of free-board from the water surface to the top of the roof slab. The entrance wall, or inlet structure, consists of a lower conduit, 10 ft 8 in. wide by 12 ft 3 in. deep, which extends the full width of the four basins, and an upper channel which distributes the water across the width of each basin by means of vertical slot openings in its inside wall. At two points, one between each pair of basins, there is a junction chamber which allows the water to flow up from the lower conduit into the upper channel or inlet of each respective basin. Each inlet channel entrance is provided with a 9 by 12-ft. sluice-gate. The inlet channel and slotted openings were constructed as nearly as possible similar to the most favorable inlet design obtained in the scale model tests previously mentioned.

The water distributed across the north end of each coagulation basin by the slotted openings in the upper entrance channel is directed straight through the basin by straight vanes or by training-walls about 29 ft long. The water flows at a velocity of 2 to 3 ft per min for the 340 ft to the south end of the basins, where it is taken off over a weir. The weir is normally submerged about 1½ ft, into a collecting conduit, 11 ft 6 in. wide by about 10 ft deep. The collecting conduits flow together at two points into junction chambers which admit the water to the Filter Building. Each collecting channel outlet is provided with a 9 by 12-ft sluice-gate for shut-off purposes. Beneath the collecting conduits is a by-pass conduit, 11 ft wide by 6 ft 6 in. deep, extending the full width of the basins, through which water may be by-passed directly from the mixing chamber to the filters. This conduit is also an equalizer of the flow from basins to the two filter influent mains. At the outlet weir there is a skimmer baffle and trough of somewhat unusual design, so constructed that, during short periods of high-water level in the basins, the collected scum will flush off into the drain. The skimmer trough acts also as an overflow.

To facilitate cleaning, the bottom of each basin is shaped into three shallow hoppers, with a sump at the center of each. A hydraulically operated 24-in. mud valve is operated on the basin drain line in this sump. The valves on the pressure lines for operating the mud valves are grouped in the two, outlet, junction-chamber, gate-houses. One basin is equipped with a perforated belt line of small pipe for flushing the floor during cleaning. If this proves efficient the other basins will be so equipped. A water main with hose connections is provided in each basin for use in cleaning.

Structurally, the basins are of simple reinforced concrete design; the side walls and division walls are of the cantilever type; the floor is of the flat-slab type; and the roof is a flat slab supported by cylindrical columns 30 in. in diameter, spaced 30 ft from center to center both ways.

The roof-slab spans are 30 ft square between intermediate panels; the end panels have reduced spans in the direction in which the continuity is broken by walls and expansion joints. Since flat slabs of 30-ft span are quite close to the maximum in common use, the analytical design was checked by a mechanical instrument simulating a model of the structure.

The inlet and outlet wall structures are of somewhat special construction, as typical of such features: Inlet and by-pass conduits are designed as rigid monolithic structures; the slotted inlet wall, subject to little horizontal load, is designed as a curtain-wall or baffle, tied at the top, bottom, and ends; the outlet weir wall is a cantilever; and the junction chambers have horizontally spanned walls.

The roof of the basins is covered with 2 ft of fill, consisting of 6 in. of pea gravel and 18 in. of earth. Over each junction chamber is a gate-house superstructure in which the 9 by 12-ft sluice-gates are hoisted by electricity and stored. The part of the basins above the general finished grade of the site (Elevation 130.0) is faced with limestone to match the building group, and the curb wall retaining the basin roof fill is surmounted by an ornamental iron fence with stone posts.

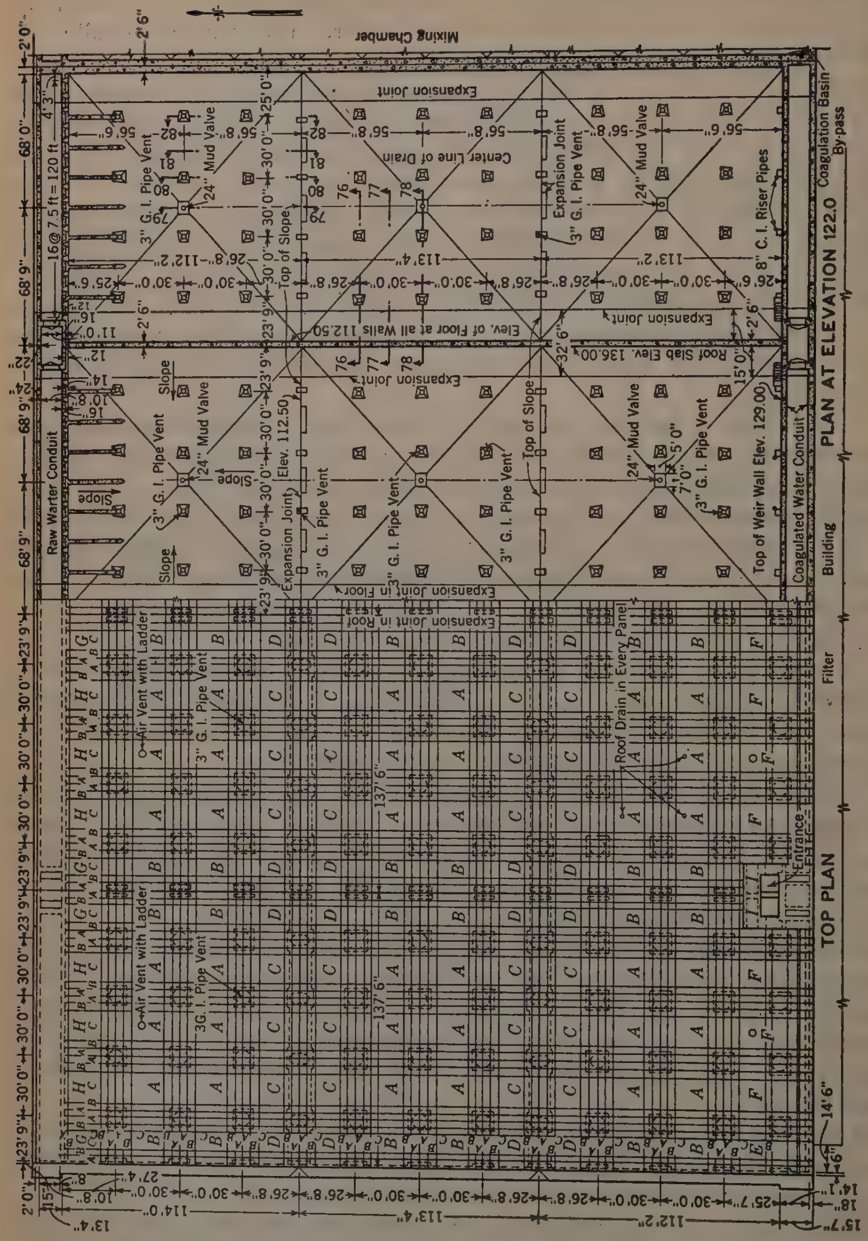

FIG. 7.—GENERAL PLAN OF SETTLING BASIN

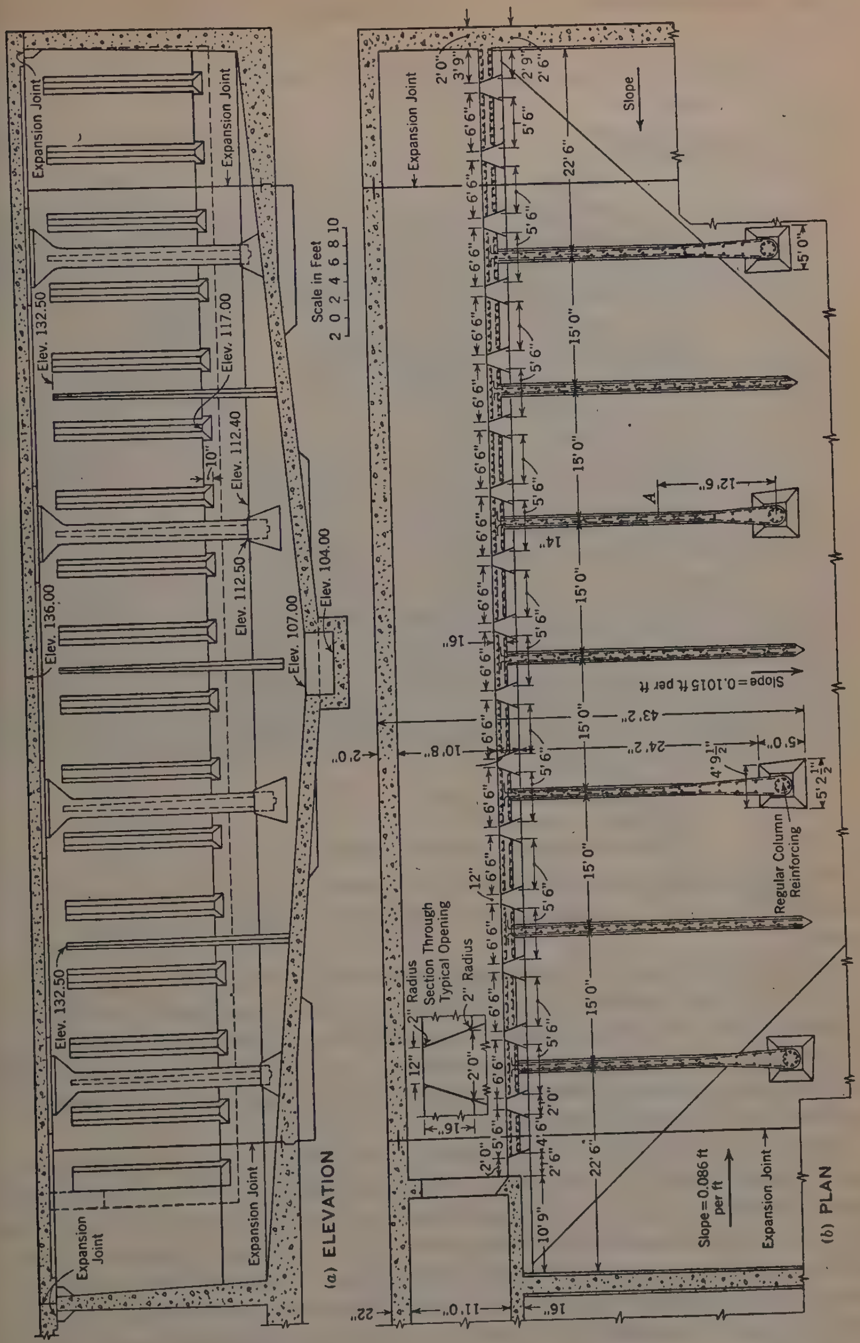

FIG. 8.—PLAN AND ELEVATION OF SETTLING BASIN INLET

The following data will serve as a summarized description of the settling basin:

Elevations, in Feet, above Detroit City Datum:	
High water	131.0
Normal water	130.6
Bottom Slab:	
At wall line	112.5
At edge of sumps	107.0
Crest of outlet weir	129.0
Volumes, in Cubic Feet:	
Water in each basin (Elevation 112.5 to Elevation 131.0)	844 770
Sludge hoppers, below Elevation 112.5	86 675
Total capacity in four basins:	
With water at Elevation 131.0	3 725 780
With water at Elevation 130.5	3 634 460
Number of basins	4
Time of detention at 306 mgd capacity, in hours	2
Velocity of flow through the basin, in feet per minute, at 306 mgd capacity	2.8
Velocities, in Feet per Second, at 306 mgd Capacity:	
Entrance velocity (normal water level)	1.0
Outlet velocity over weir (normal water level)	0.6
Maximum velocity in basin outlet channel	1.0
Turbidity of settled water, in parts per million.	10 to 15
Dimensions, in Feet:	
Length of flow in basin	340.0
Width of one basin	135.5
Effective water depth, one basin	18.0 to 18.5

Slotted entrances and 29-ft guide-vanes constitute the baffling system; the outlet of the basin is operated as a submerged weir; and the basins are covered.

Filters.—The group of sixty-eight filter units is adjacent to and centers on the south wall (which is the outlet end) of the settling basins. The filters are arranged in four rows along two parallel pipe galleries with concrete influent conduits connected by cross-conduits to the outlet chambers of the settling basins. The top slab of the influent conduits forms the operating floor between the rows of filters.

The interior of a filter unit showing the wash troughs and the under-drain system before the filtering material is placed, is shown in Fig. 9. The filters are of the front gullet type, with longitudinal wash troughs of cast iron and perforated cast-iron pipe under-drains. The under-drain laterals are connected to a central manifold cast in the concrete bottom of the filter. The ends of the laterals at the side walls of the filter unit are connected by 4-in. cast-iron headers parallel to the walls to eliminate dead ends and to assure more uniform distribution of flow in the under-drain system. These wall headers are perforated also to give a slight excess of wash water along the filter walls, during washing.

The filtering medium consists of an 18-in. depth of specially selected gravel and 20 in. of silica sand. The gravel grades in size from 3 in., maximum diameter at the bottom, to $\frac{1}{16}$-in. particles at the top of the gravel layer. The sand is 95% pure silica having an effective size of 0.5 mm and a uniformity coefficient of 1.3; 2% of it is composed of particles larger than 1 mm. The sand finer than 0.3 mm was washed out and is disregarded in computing the effective size and uniformity coefficient.

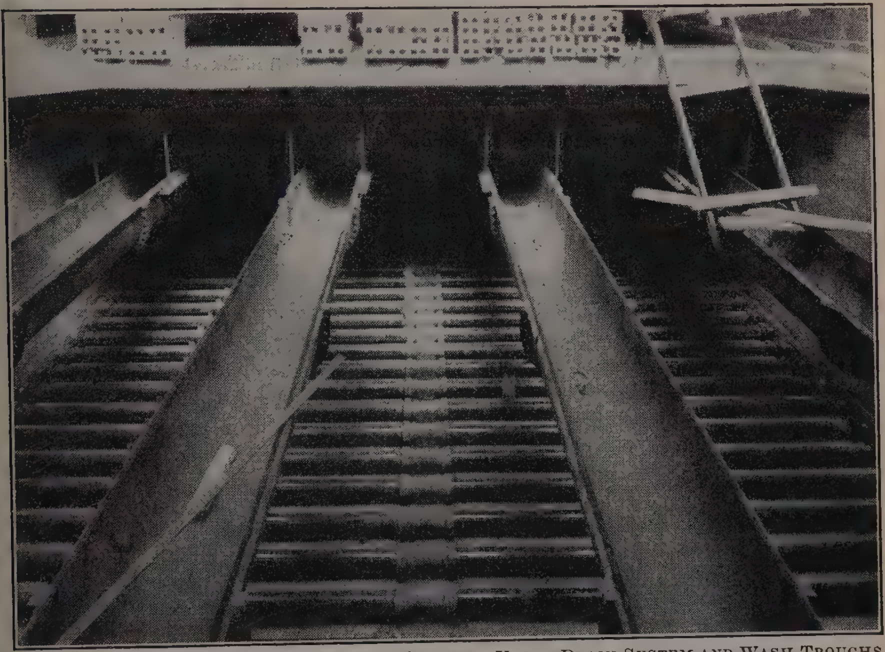

FIG. 9.—VIEW OF INTERIOR OF FILTER, SHOWING UNDER-DRAIN SYSTEM AND WASH TROUGHS

While it is filtering, water normally stands within 6 in. of the top of the filter walls, making about 7 ft of water depth on the surface of the sand bed. In washing, the sand expands 50% and rises to the bottom of the lower ends of the wash troughs.

A typical section of the pipe galleries and the piping for a typical filter unit is shown in Fig. 10. All header conduits are of concrete and are arranged to form the floor of the pipe gallery as well as the filter operating floor above. The filter connections are made with cast-iron fittings and flanged valves of the following sizes (in inches):

Influent	36
Effluent	24
Wash water	24
Sewer	30
Rewash	8

All valves are of the cast-iron, bronze-mounted, double-disk type, actuated by hydraulic cylinders operated from a marble-faced operating table or cabinet located on the filter operating floor at the front of each filter. All hydraulic cylinders are bronze-lined throughout, including the cylinder heads and pistons, and all pressure piping to the valve cylinders is of seamless copper tubing.

Each filter is equipped with a rate-of-flow controller of the Venturi type and a loss-of-head gauge, both actuating an indicating and recording gauge head mounted on the filter operating table above. The rate controllers are arranged for master-control setting or for individual setting as desired. The pipe galleries are heated by steam radiators and ventilated by a line of steel grating along each side of the operating floor of the filter.

The wash-water supply system consists of two pumps with capacities of 6 000 gal per min each, two pumps with capacities of 8 000 gal per min, located in recesses off one pipe gallery, and two 50 000-gal tanks in the cross-monitors of the Filter Building. The pumping capacity is sufficient for washing filters continuously, one at a time, while the tank capacity is sufficient for one single-filter wash. The depth of the wash-water tank and the rate of filter washing are indicated on a large illuminated gauge near the center of each operating gallery.

The following data will serve as a summarized description of the filters:

Rate of Filtration, in Million Gallons per Day, per Acre:	
Maximum	180
Normal	160
Number of filter units	68
Capacity of a filter unit (maximum rate), in million gallons daily	4.5
General Dimensions, in Feet:	
Width of sand bed	27.00
Length of sand bed	40.33
Depth of filter box	11.0
Clear distance between troughs	4.44
Height, bottom of wash-water tank above sand surface	35.5
Detail Dimensions, in Inches:	
Diameter of connections:	
Influent	36
Effluent	24
Wash water	24
Drain	30
Re-wash	8
Thickness of sand bed	20
Gravel:	
Thickness of layer	18
Maximum size	3
Minimum size	$\frac{1}{16}$
Height, surface of sand to crest of wash troughs	40

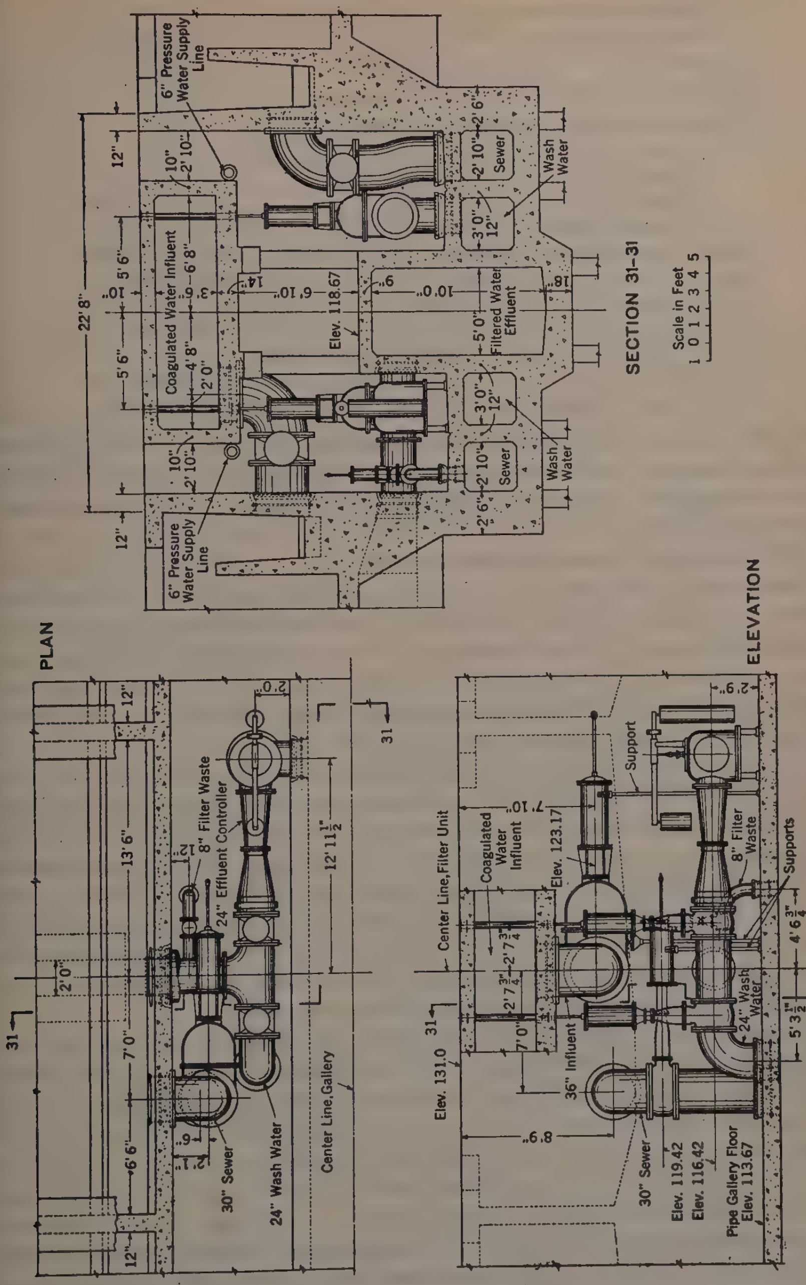

FIG. 10.—PIPE GALLERY DETAILS, FILTRATION PLANT

Elevations, in Feet, above Detroit City Datum:	
Bottom of filter	120.0
Water surface on filters	130.5
Effective size of sand, in millimeters	0.5
Uniformity coefficient of sand	1.3
Under-Drains (Dimensions, in Inches):	
Laterals, size	4
Laterals, spacing, center to center	12
Orifices, size	0.504
Orifices, spacing, center to center	6
Ratio of orifice area to sand area (× 100)	0.277
Rate of washing, in inches per minute	30 to 39
Tank capacity for wash water, in gallons (net)	95 000
Pump capacity for wash water, in gallons per minute	28 000
Percentage of wash water used (average)	1.3

The strainer system is composed of perforated cast-iron pipe, with perforations brass bushed. The wash gullets are at the front of the filters, and the sand is not agitated during the washing process except by the high velocity of the wash water.

Filter Building.—The filter group housed in a single-story building with two longitudinal monitors, over the filter operating galleries, and two cross-monitors in which the wash-water tanks and air-conditioning and heating equipment are placed. This building is of concrete-encased, structural-steel frame construction, with reinforced concrete roof of the ribbed-slap type. The curtain-walls are of hollow tile faced with buff terra cotta on the interior and chat-sown Indiana limestone on the exterior.

The concrete ceiling and columns are painted a light cream color. The operating floor is finished with 6-in. square, red, quarry tile, laid in black mortar, with borders and spill-rail curbs of dark green terrazzo. A railing of ornamental iron with an oak top rail is along each side of the operating galleries. All ornamental iron work, steel floor gratings, ventilator grilles, and steel sash and door framing are painted a dark green. The roof is insulated with a 1½-in. layer of pressed cork on top of which is standard 4-ply tar and gravel roofing. Large skylights of the vault type in the roof over the filters augment the light from the gallery monitors.

Heating System.—Considerable attention was given to the heating and ventilating of the Filter Building. Tests made in the existing Water-Works Park Plant indicated considerable variation in temperatures throughout the building and a large consumption of heat to maintain an average of 50° F during zero weather. Condensate was seen to collect on the steel work, roof slabs, and window sash in the spring and fall when the building was not heated. With this example to observe and test, it was expected that an improved heating and ventilating system could be devised for the Springwells Filtration Plant. The uncertain heat loss to the open water surface of the filters complicated the problem considerably. Attempts to measure this loss failed, due to lack of instruments for measuring such a small temperature change. A heat transfer coefficient of 2.00 was taken as the most probable for heat loss to an open water surface. In collaboration with experienced heating and ventilating consultants, the following outline of a heating system for the Springwells Filter Building was developed.

Heating.—The pipe galleries were to be heated by direct radiation; and the main building (except one room), by re-circulated air. The controlling temperatures, in degrees Fahrenheit, were:

Maximum average, controlled to	70
Minimum, average	50
Maximum allowable variation in working area	20
Maximum temperature of heated air	90

Ventilation.—The specifications required four changes of air per hour, re-circulated, with one change of fresh air admitted and tempered. Stale air was to be exhausted from the floor, at the ends of the pipe galleries, at the rate of ½ air change per hr. A probable air leakage of ½ air change per hr from the building was expected, due to maintaining inside the building, by means of the fans, a slightly higher pressure than normal atmospheric pressure. Re-circulated air was to be taken from the main roof at the center of the filter group and returned at the four intersections of the operating galleries, and at the cross-galleries. Maximum permissible air velocities, in feet per minute, were established as follows:

In air ducts	600
At outlets of fans	1 600
Through heaters	1 000

The total heat required to maintain the main part of the building at an average temperature of 50° F, with zero temperature outside, and with water at 32° F in the filters, was computed as 6 846 900 Btu per hr. This heat loss was apportioned as shown in Table 2. The loss in the pipe galleries could be estimated only approximately. Considering the exposed area of pipes as condenser surface and estimating 1 air change per hr due to convection currents, a total of 587 600 Btu per hr was computed as necessary to keep the pipe galleries above 50° F, with water at 32° F in the plant, and the main building at 50° F.

TABLE 2.—Heat Required to Maintain the Filter Building at 50 Degrees Fahrenheit, with 0 Degrees Fahrenheit, Outdoors

Surface	Area, in square feet	Heat, in British thermal units per hour	Surface	Area, in square feet	Heat, in British thermal units per hour
Glass	7 163	526 690	Skylights	9 360	598 000
Walls	33 681	450 510	Water	80 784	2 586 000
Roof	101 744	874 800	Floor over conduits	21 960	131 400
For 1 air change (1 608 000 cu ft) per hr					1 679 500

The heating surface required was computed on the basis of 5-lb steam pressure in the heaters and radiators. An allowance of a 2-lb pressure drop in the steam lines between the source of the steam (the low-pressure side of the turbines in the generator room of the power plant) and the farthest heater, governed the sizes of steam pipes.

The heating equipment consisted of the following principal units, complete with accessories:

Four blower fans of 33 500 cu ft per min capacity of air at 70° F against a back pressure of ⅞ in. of water.
Four batteries of cast-iron heaters having 2 064 sq ft of heating surface each, for re-circulated air.
Four batteries of cast-iron heaters having 408 sq ft of heating surface each, for tempering fresh air.
Four dampered louvers for fresh-air intakes.
Four distributing louvers or diffusers for re-circulated air.
Four exhaust fans of 4 000 cu ft per min capacity of air at 70° F against a back pressure of 0.3 in. of water.
Twenty-eight radiators in pipe galleries, each of 126-sq ft heating surface.
Two vacuum return pumps for pipe-gallery radiation, each of capacity ample for 5 000 sq ft of radiation.

The re-circulating fans and air heaters are installed in the space beneath the wash-water tanks in the cross-monitors of the building. Condensed steam returns by gravity from the air heaters to a vacuum return pump in the basement of the plant office, which is on the route back to the boilers.

The steam main to the air heaters in the monitors is installed on the roof of the Filter Building with the air-recirculating duct. The steam supply lines to the pipe galleries are placed along the walls of the galleries. All steam lines and return lines are insulated with 85% magnesia pipe covering, re-canvassed. An exposed part of the steam main on the roof is also covered with a steel sheath. The heating system was installed during the time of interior finishing and was completed soon after the building was constructed.

Office and Laboratory Building.—This building was constructed complete under one general contract, including heating, ventilating, plumbing, electrical work, and all other trades. Its construction was typical of buildings of this nature.

The first floor contains offices in front for the filter plant superintendent, the power and pumping plant superintendent, the clerical force, and a conference or receiving room. The rear of the first floor contains locker rooms, shower rooms, and toilets for the plant workmen and operators, as well as a time office and storage rooms. The second floor contains the filter plant laboratory, with offices for the chief chemist and bacteriologist.

Weir Chamber.—It was required that the effluent water level be maintained at a height sufficient to submerge the filter-effluent connections and effluent-main conduits and to operate the shunt system.[4] For this purpose, a chamber is provided, at the outlet ends of the filtered-water, effluent-main, conduits, which contains a filter-seal weir with crest at Elevation 118.00 and a reservoir weir, in two sections, with crest at Elevation 119.50, as shown in Fig. 11. This chamber is of reinforced concrete construction. In addition to its operating function it serves as a foundation for the plant office as well as providing storage space for stop-logs, spare parts, castings, etc. The two sections of reservoir weir are built diagonally to gain length and also to converge the flow from the filter-seal weir to the entrance of the conduits leading directly to the high-lift pumping plant. A central division wall is provided for use in shutting off one-half the plant. Openings in this division wall and in the reservoir-weir walls (filled by stop-logs) are provided for flexibility in by-passing or running under special operating conditions.

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[4] "Shunt System of Operating Filtered Water Reservoirs," by E. A. Hardin, *Engineering News-Record*, Vol. 103, No. 26, December 26, 1929, p. 1011.

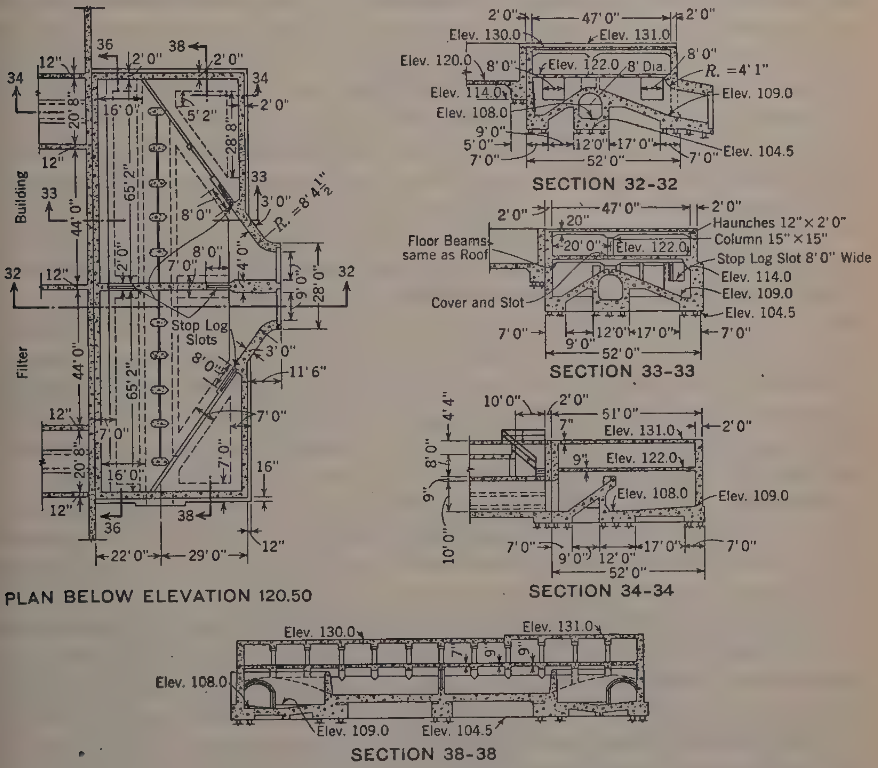

FIG. 11.—SECTIONS OF WEIR CHAMBER

Two 9 by 9-ft conduits which run from the weir chamber to the west end of the high-lift pumping plant, are designed to carry water direct to the pumping plant. An 8 by 8-ft conduit conveys water from the south end of the weir chamber to the filtered-water reservoir. The conduits to the pumping plant take the water that overflows the filter-seal weir, while the reservoir conduit takes the water that overflows the higher reservoir weir.

The Shunt System.—The shunt system is a method of operating the high-lift pumping plant so that about three-fourths of the total pumpage is drawn directly from the filter plant at about $1\frac{1}{2}$ ft above the level of the effluent-seal weir, while the remainder, which is taken on the daily peak, is drawn from the filtered-water reservoir at a level that may vary with the reservoir stage, from Elevation 120.0 to Elevation 108.0 each day. Thus, considerable head on the suction side of the pumps is conserved for most of the water pumped. This is accomplished by dividing the pumping units into two groups with a division wall in the suction gallery of the pumping plant. A small group at the remote end from the filter plant, comprising about one-fourth of the high-lift pumping capacity, draws from the reservoir through Gate Chamber No. 1 and reservoir-outlet conduits connected to the east end of the pump suction gallery, while the remainder of the pumps draw directly from the filter plant, *via* the weir chamber.

The weir chamber effects the division of the water at the filter plant by a secondary weir, called the reservoir weir, the crest of which is high enough above the seal weir so that the entire flow of filter effluent may pass over it without overtopping the reservoir weir. The direct flow to the high-lift pumping plant is taken off between these weirs. Thus, the entire capacity output of the filter plant may pass directly to the high-lift pumps without affecting the reservoir levels at all. If the pumps taking suction direct from the filter plant do not require the full filter output, the effluent level rises until it overtops the reservoir-weir crest at Elevation 119.5, and the excess is discharged to the reservoir. During peak pumpage hours, when the output of the high-lift plant is more than that of the filter plant, pumps at the remote end are placed in service as required to meet the demand, and the pumpage in excess of the filter output rate is taken from the reservoir.

For convenience in operating the pumps, a water-level gauge is provided in the weir chamber between the two weirs and a control level of about Elevation 119.50 is maintained at this point. If this water level rises above Elevation 119.50, it is known that water is flowing into the reservoir and that, if possible, additional work by the pumps west of the division wall is permissible and less on the east side of the division wall is in order. If the control water level falls to Elevation 119.0, or below, less pumping west of the division wall, and more, east of the division wall, is desirable. To safeguard the operation of the pumps and to insure suction water from the reservoir to all pumps in the high-lift plant, flap-gates are provided in the suction-gallery division wall which are opened by the pressure from the reservoir-water side at any time the water level on the filter-plant side is below that on the reservoir side of this wall.

From the foregoing it is seen that by the shunt system the main flow of water from the filter plant is diverted directly to the high-lift pumps at a normally constant high level, and the reservoir is on a secondary loop, or shunt, feeding a small group of pumps, normally isolated as to suction from the remainder of the station. Experience with the shunt system at the Water-Works Park Station (see Fig. 1) has shown that it causes no inconvenience whatever in the pump operation. The pumps in the reservoir group, as a rule, are put in service only in peak hours and little concern is given the shunt system, which works automatically.

TABLE 3.—COMPUTED VELOCITIES, HEAD LOSSES, AND WATER-SURFACE ELEVATIONS

Item No.	Point	AVERAGE CAPACITY (210 MGD)			MAXIMUM CAPACITY (306 MGD)		
		Velocity in feet per second	Loss of head, in feet	Elevation, in feet	Velocity in feet per second	Loss of head, in feet	Elevation, in feet
	(*a*) ENTRANCE TO MIXING CHAMBERS						
1	Elevation above Detroit City Datum, in feet	....	......	131.13	....	......	131.60
2	Entrance gates (two, 6 by 10 ft)	2.70	0.11	......	3.95	0.25	......
3	Mixing chamber	0.29	0.01	......	0.43	0.02	......
4	Intermediate gates (four, 6 by 10 ft)	1.35	0.04	......	1.97	0.09	......
5	Outlet gates (three, 6 by 10 ft)	1.80	0.05	......	2.63	0.11	......
6	Conduit to coagulation basins	1.70	0.12	......	2.48	0.28	......
7	Entrance to coagulation basins	0.75	0.02	......	1.09	0.04	......
	(*b*) INLET END OF COAGULATION BASINS						
8	Elevation above Detroit City Datum, in feet	....	......	130.59	....	......	130.72
9	Coagulation basins	0.03	0.00	......	0.05	0.01	......
10	Basin outlet weir	0.65	0.03	......	0.95	0.06	......
11	Outlet channel and 9 by 12-ft gate	0.75	0.03	......	1.09	0.05	......
12	Junction chamber and settled water conduit to gallery (12 by 20 ft)	0.68	0.01	......	0.99	0.03	......
13	Settled water conduit to Gallery 2 (6 by 19 ft)	0.7	0.00	......	1.05	0.01—	......
14	Filter influent header	0.7 to 0	Slight regain	......	1.0 to 0	Slight regain	......
15	Filter inlets	0.77	0.02	......	0.99	0.04	......
	(*c*) FILTERS						
16	Elevation above Detroit City Datum, in feet	....	......	130.50	....	......	130.50
17	Filters (available operating head)	....	9.95	......	....	9.11	......
18	Filter effluent piping and wide-open rate controller	1.72	0.63	......	2.22	1.03	......
	(*d*) NORMAL GRADIENT IN UP-STREAM END OF EFFLUENT HEADER						
19	Elevation above Detroit City Datum, in feet	....	......	119.92	....	......	120.36
20	Filter effluent main	0.23 to 3.25	0.26	......	0.36 to 4.83	0.55	......
	(*e*) ABOVE EFFLUENT WEIR IN WEIR CHAMBER						
21	Elevation above Detroit City Datum, in feet	....	......	119.66	....	......	119.81
22	Effluent weir	1.63	0.16	......	3.09	0.31	......
	(*f*) BELOW EFFLUENT WEIR IN WEIR CHAMBER						
23	Elevation above Detroit City Datum, in feet	....	......	119.50	....	......	119.50
24	Conduits to high-lift pumping plant (two, 9 by 9 ft)	2.0	0.10	......	2.9	0.21	......
25	Entrance to 50-mgd pump suction chamber (6 by 6-ft sluice-gate)	2.15	0.20	......	2.15	0.20	......
	(*g*) HIGH-LIFT PUMPS ON DIRECT DRAFT						
26	Elevation above Detroit City Datum, in feet	....	......	119.20	....	......	119.09
	(*h*) NORMAL HIGH-WATER LEVEL IN RESERVOIR						
27	Elevation above Detroit City Datum, in feet	....	......	119.00	....	......	121.00
	(*i*) LOW-WATER LEVEL IN RESERVOIR						
28	Elevation above Detroit City Datum, in feet	....	......	103.00	....	......	103.00

Hydraulics of Flow Through Plant.—The computed velocities, head losses, and water-surface elevations at governing points throughout the plant are given in Table 3 for both a maximum-capacity flow of 306 000 000 gal daily, and an average-capacity flow of 210 000 000 gal daily. For these computations it is assumed that the water-surface elevations in the filtered-water reservoir are less than 119.5. When the reservoir is at higher stages, up to its normal high-water level of Elevation 121.00, the filter effluent levels will be raised correspondingly (except for the slight effect of the greater submergence of the effluent weir), thus reducing the available operating head of the filters at times of high water in the reservoir.

Under-Drainage System and Main Drains for Plant.—The entire sub-grade area of the filtration plant and reservoir is provided with drains for controlling the elevation of the ground-water under the plant. The drains are of 6-in. vitrified sewer pipe laid with open joints in shallow trenches at the surface of the sub-grade and surrounded by open gravel. They are spaced approximately 7 ft from center to center, located so as to clear the piling. Under the coagulation basins and mixing chamber and under the floor-slabs of the filters there is also a layer of open gravel about 4 in. thick for distributing the water between the drains. There is no gravel layer between the drains under the reservoir. There are no drains under the wall footings and the pipe galleries. In addition to the under-drains there is also a belt drain of perforated, 12-in., cast-iron pipe laid completely around each structure at the top of the wall footings to intercept ground-water from the surrounding area.

The drainage system under the structures constituting the filtration plant is connected to the main drain from the plant by an overflow connection which discharges water when the level is above Elevation 118.5. It is expected that, usually, there will be a flow from the drains due to leakage and ground-water; but to assure a ground-water level at sufficient height to cover the timber piling a water-supply connection is provided with a float-operated valve that admits water to the drainage system when the level in it becomes less than Elevation 118.5 (which is 4 ft above the top of the highest pile).

The level in the reservoir drainage system is maintained at desired levels by float-controlled sump pumps which discharge into the sewer.

Filtered-Water Reservoir.—The reservoir for filtered water consists of two sections, each with a capacity of 20 000 000 gal, and each a rectangle, 455 ft long by 313 ft wide (interior dimensions). There will be a third section, eventually, of about the same size. Water flowing to the reservoir is distributed to the sections by a conduit, built along the north wall, which is connected to the sections through a gate-chamber in each one and also through an additional 66-in. inlet valve in the east section. In each gate-chamber are four 48-in. double-disk gate-valves, hand-operated by geared floor stands with cranks. The water may enter or leave through these valves. Flap check-valves are placed on the gate-valve openings in such a manner that the water flows in through the valves on one side of the center baffle-wall and out through the valves on the other side. By this means, water may be circulated through the reservoir sections, if desired. In the east section, the 66-in. valve serves as the inlet, and two of the 48-in. valves equipped with flap-gates serve as the outlet, to produce circulation around the baffle-wall. Figs. 12 and 13 give the general plan and sections of the existing sections of the reservoir.

The reservoir is constructed of reinforced concrete of standard design. The walls are one-way slabs spanned vertically between the base and the roof. The roof and floor are of typical two-way flat-slab construction with cylindrical columns spaced 20 ft center to center both ways.

The reservoir is completely back-filled and its roof is covered with 6 in. of gravel and 18 in. of earth. It thus forms the front-yard area of the station and will be landscaped and planted. Water is drawn from the reservoir to the pumps in the east end of the high-lift pumping plant through two, 8-ft square, concrete conduits running from Gate Chamber No. 1 to the east end of the suction gallery in this pumping plant.

CONSTRUCTION

General Plan and Administration.—In constructing the Springwells Filtration Plant and Reservoir, the Board of Water Commissioners acted somewhat in the capacity of a general contractor. Most of the equipment, valves, sluice-gates, piping, and castings were purchased separately, and contracts for the construction work and installation were let to building contractors and trades, this equipment, piping, castings, etc., being furnished. The reservoir was constructed under two contracts. The filtration plant was constructed under eight seperate major contracts: (1) Excavation and piling; (2) substructure (mostly heavy concrete construction); (3) superstructure of Filter Building; (4) electrical work in Filter Building; (5) plumbing, heating, and ventilating work in Filter Building; (6) Chemical Building, including alum-handling equipment; (7) masonry facings of coagulation basins and mixing chamber and the construction of the superstructures of the drainage pump house and two gate-houses on the coagulation basins; and (8) complete construction of the Office and Laboratory Building.

The operating tables in the filtration plant, the chlorinators and chlorine piping, the chlorine scales, chlorine hoist, pressure water pumps and sump pumps, drainage pumps, filtered water meters, wash-water pumps, wash-water meters, and the wash-water pipes were furnished and installed under separate equipment contracts, let directly to the various equipment manufacturers.

The subdivision of the work resulted in smaller contracts and the letting of work directly to the proper trades. This eliminated considerable administration cost from the actual contracts, but required considerable administrative work on the part of the Engineering Division of the Board (which handled the work) with attendant expense reflected in the engineering cost figures, as shown in Table 4 discussed subsequently. It is believed that the savings in contract costs effected by this procedure considerably more than offset the increased administrative and engineering expense involved. Considerable saving in contract costs was obtained also by fully and completely detailing the contract drawings upon which bids were based, thus eliminating guesswork on the part of the bidders and resulting in low and close bidding. Dividing the work in this manner expedited the construction since construction was

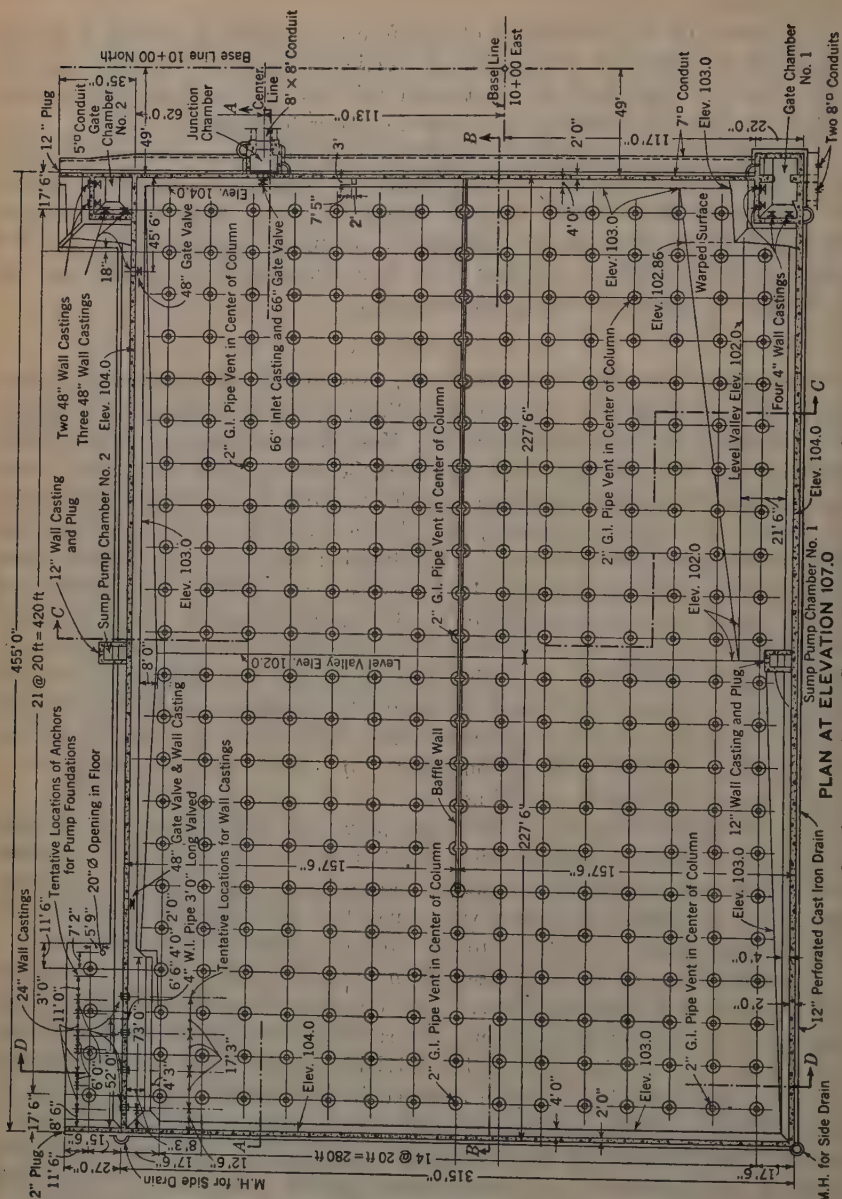

FIG. 12.—PLAN OF FLOOR-SLAB, FILTERED WATER RESERVOIR NO. 2

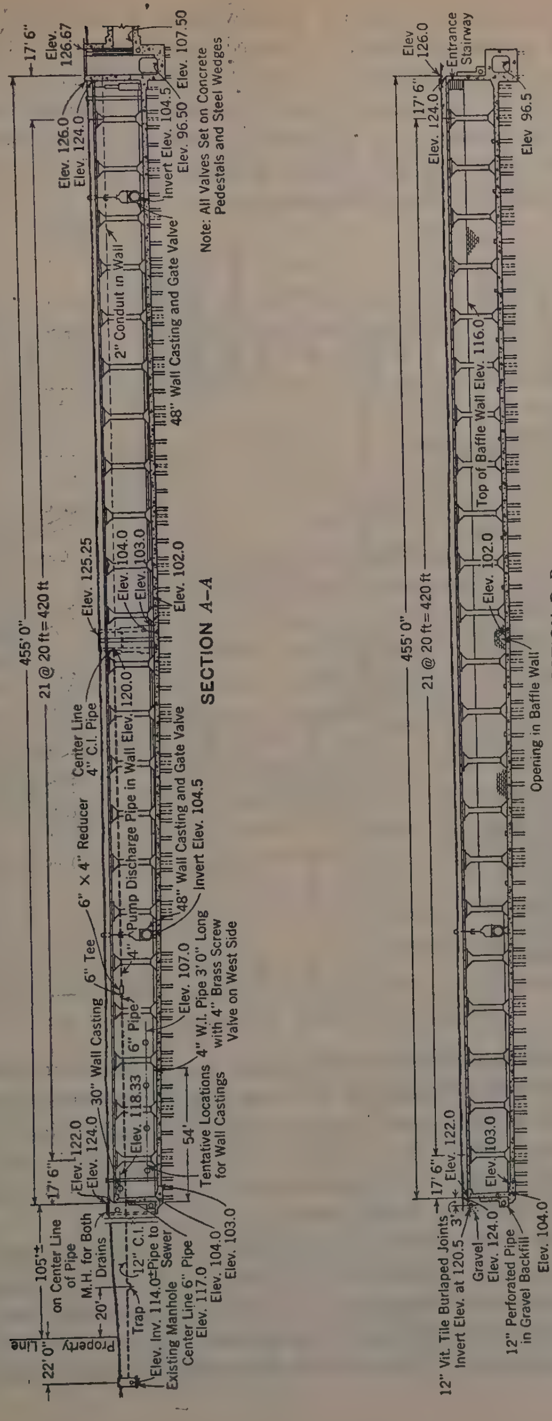

FIG. 13.—GENERAL SECTIONS, FILTERED WATER RESERVOIR NO. 2

begun on the foundations while the plans were being completed for the superstructures and subsequent work. To schedule and expedite the various purchases and contracts properly, in order to prevent delays and to preserve harmonious working conditions where a number of different contracts were under way on the limited area of the site, involved careful attention both to the preliminary planning and to the progress of the work. It also added greatly to the duties of the engineering force.

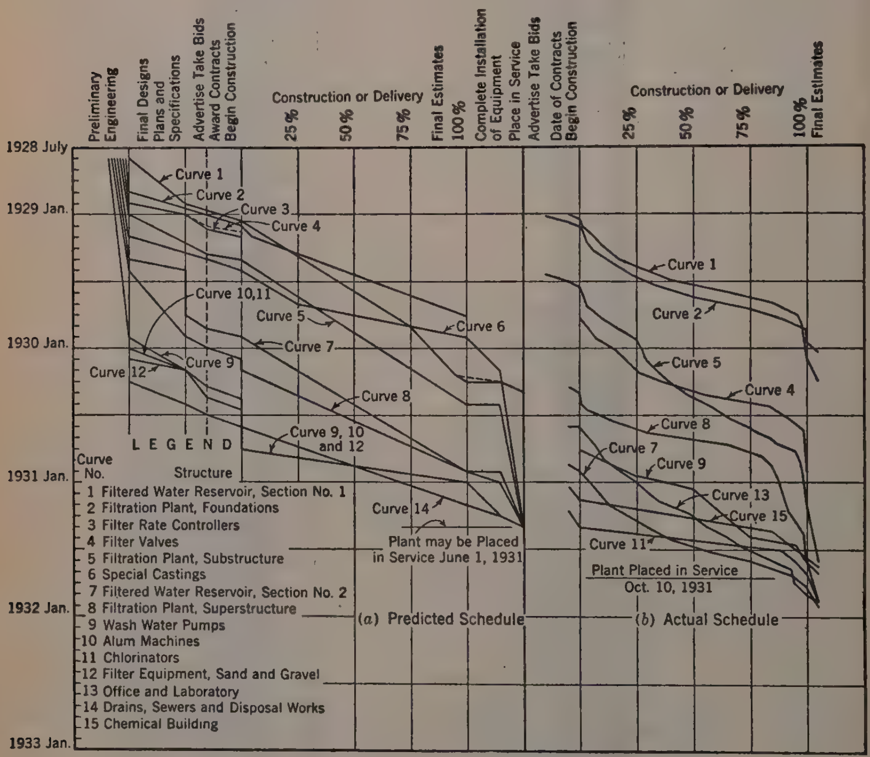

FIG. 14.—PROGRESS OF CONSTRUCTION

Progress Schedule.—Before any purchase was made or contract prepared, a preliminary progress schedule of the complete project was made as shown in Fig. 14(*a*) by which it was estimated that the plant might be placed in operation by June, 1931. This schedule, of course, could not be followed exactly, but served as a guide in preparing the work. The actual construction schedule is similarly shown in Fig. 14(*b*).

The construction was completed to the point at which operation of the filter plant would have been possible July 1, 1931, if the water demand had required it. However, since the drop in water demand resulting from the general recession in business activity made the operation of the plant less urgent at the time, and since several miscellaneous items of work were desirable and more economical to be done before the plant was put in service, the actual operation was begun on October 10, 1931, about four months later than scheduled.

Construction Methods.—There was nothing particularly special or peculiar about the construction of this project. Standard first-class construction methods were used throughout. Most of the excavation was by drag-line excavators and shovels. Piles were driven by ordinary wooden skid drivers. Concrete was mixed in a central mixing plant for each job and transported to the forms by chutes, belt conveyors, and industrial train, each contract making use of a different transporting method. The mixers were all of about 1 cu yd capacity. The superstructure work was typical of such construction. The major part of the structural work, except the excavation and pile-driving for the Filter Plant, was done by two contracting firms, the W. E. Wood Company and the Bryant and Detwiler Company, of Detroit, Mich. The Whitney Brothers Company, of Duluth, Minn., did the excavation and piling work for the filtration plant.

The construction photographs, Figs. 15 to 18, inclusive, show the magnitude and type of the construction work. They were taken from the same point (the top of the concrete mixing plant) and show the filtration plant in successive stages of construction.

The principal problems of construction arose from the magnitude of the project and the extent of the area covered by the plant structures, and were mainly problems of transportation. Approximately 18 acres were occupied by the actual structures themselves and large volumes of materials had to be handled for considerable distances over this construction area while many large pipe castings, valves, and pieces of equipment had to be installed as the work progressed.

A general idea of the magnitude of the work may be obtained from the following summary of the approximate quantities of the principal construction materials that went into the Springwells Filtration Plant and Reservoir:

Excavation, in cubic yards	420 000
Timber bearing piling, in linear feet	1 500 000
Concrete, in cubic yards	120 000
Reinforcing steel, in tons	7 200
Concrete forms, in square feet	2 000 000
Structural steel, in tons	1 120
Ornamental iron, in tons	70
Iron pipe and castings, in tons	2 000
Gravel fill and drainage material, in cubic yards	15 000
Tile under-drains, in linear feet	56 000
Stone masonry, in cubic feet	42 000
Brick and tile masonry, in cubic feet	77 000
Terra cotta, in square feet	33 000
Steel sash, in square feet	11 000
Metal doors, in square feet	1 000
Tar and gravel roofing (squares)	1 250
Skylights, (vault type), in square feet	11 000
Cork insulation, in square feet	134 000
Copper roofing and flashing, in square feet	35 000
Terrazzo and ground concrete floors, in square feet	30 000
Quarry tile floors, in square feet	22 000

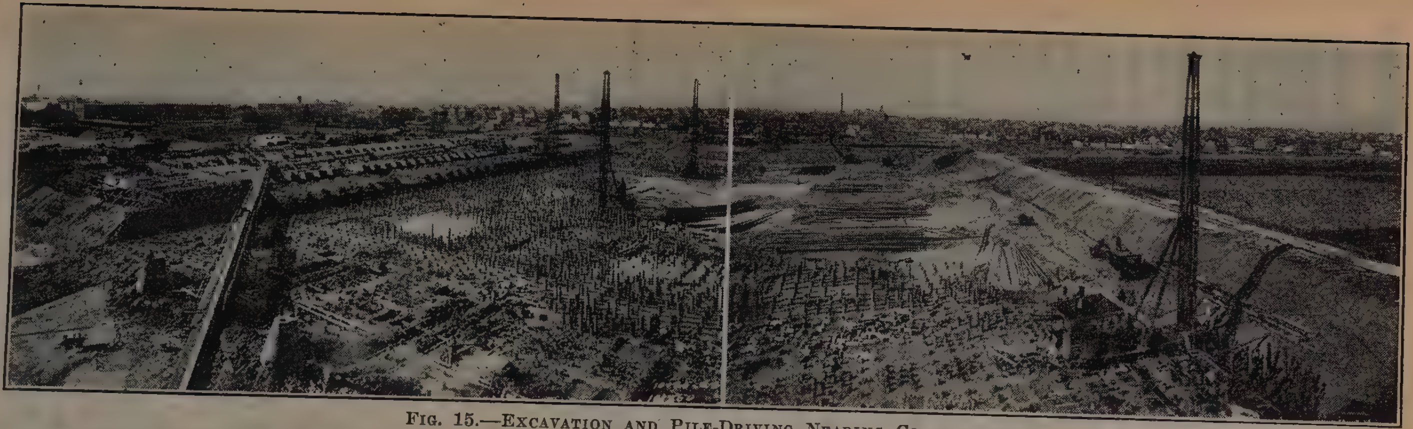

FIG. 15.—EXCAVATION AND PILE-DRIVING NEARING COMPLETION

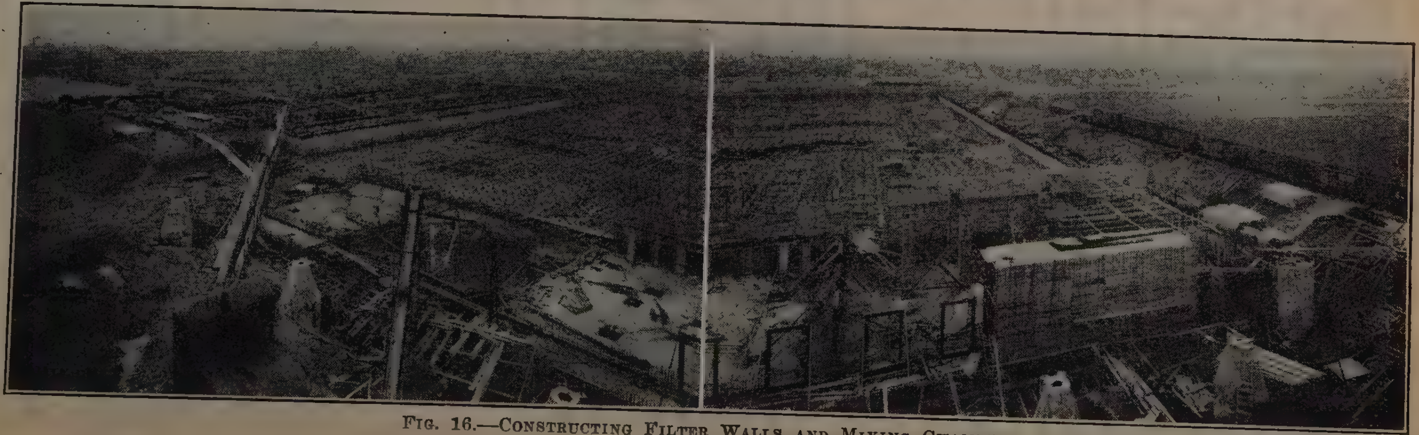

FIG. 16.—CONSTRUCTING FILTER WALLS AND MIXING CHAMBER

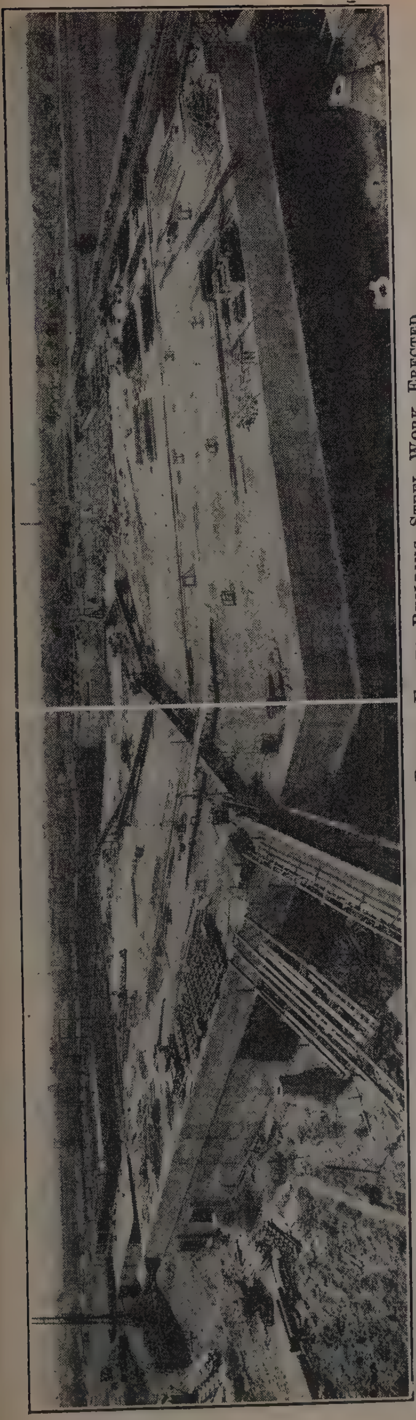

FIG. 17.—EXTENDING SETTLING BASIN; FILTER BUILDING STEEL WORK ERECTED

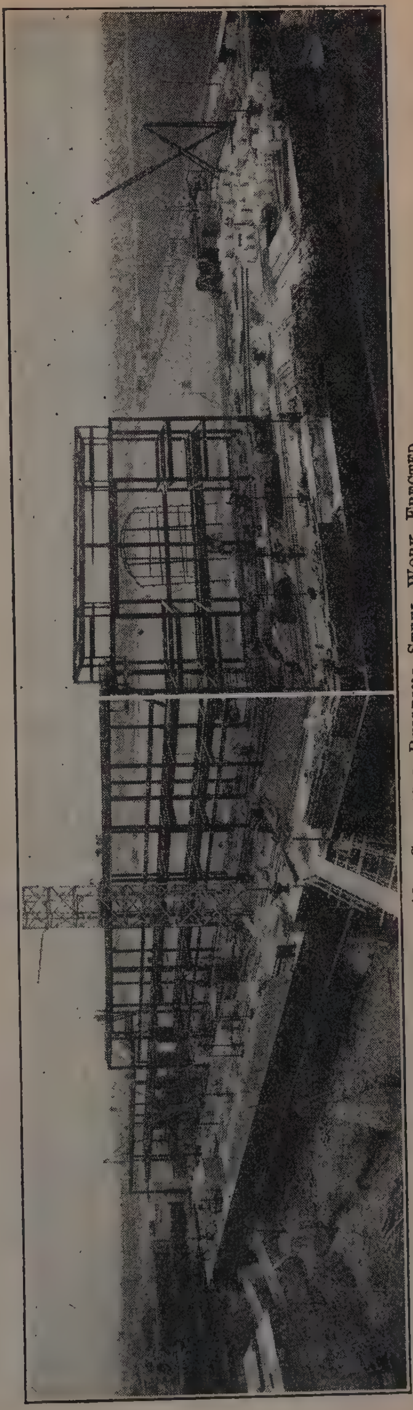

FIG. 18.—CHEMICAL BUILDING STEEL WORK ERECTED

TABLE 4.—Cost of Springwells Filtration Plant

Item No.	Units constructed	Construction Cost: Actual	Construction Cost: Adjust to 1913 costs by Index Number	Date of receipt of bids	Cost index based on the year 1913 as 100
	(*a*) Filtered-Water Reservoir				
1	Section 1: Contract	$541 478	$258 585	January 16, 1929	$209.40
2	Section 1: Force account work	14 055	6 789	......	207.02
3	Section 2: Contract	347 226	176 382	December 2, 1930	196.86
4	Section 2: Force account work	16 612	9 160	......	181.35
5	Section 3: Force account work	125	74	......	170
6	Section 3: Future construction (estimated cost, including grading, planting, and roadways)	400 000	235 300	......	170
7	Sections 1, 2, and 3: Equipment, including valves, pumps, and level gauges (installation of valves included in Items Nos. 1 and 3)	56 971	27 770	May 13, 1929	205.15
	Cost of Engineering:				
8	Total, exclusive of supervision on Section 3	74 577	36 028	......	207
9	Estimated for supervision on Section 3	20 000	11 765	......	170
10	Total cost of reservoir (60 mg capacity)	$1 471 044	$761 853	......	......
11	Cost of engineering ($94 577), in percentage of total cost	6.43	......	......	......
12	Total cost per million gallons capacity (60), including engineering, incidental expense, and contingencies	$24 517	$12 698	......	......
	(*b*) Filtration Plant Structures				
	Foundation Work (Excavation and Pile-Driving):				
13	Contract	$607 035	$288 515	February 6, 1929	$210.40
14	Force account labor and miscellaneous expense on foundation work	19 692	9 512	......	207.02
	Substructure Construction:				
15	Contract	1 518 574	741 600	July 10, 1929	204.77
16	Force account labor and miscellaneous expense on substructure construction	55 680	27 449	......	202.85
	Superstructure of Filter Building:				
17	Contracts	604 754	291 982	April 30, 1930	207.12
18	Force account labor and miscellaneous expense	25 910	12 773	......	202.85
	Chemical Building, Including Mechanical Equipment, Chemical-Handling Equipment, Heating, Ventilating, Plumbing, and Electrical Work:				
19	Contract	178 100	91 577	January 27, 1931	194.48
20	Force account labor and miscellaneous expense	11 713	6 459	......	181.35
	Basin Facings and Gate-Houses:				
21	Contract	46 381	27 318	October 20, 1931	169.78
22	Force account labor	1 746	1 032	......	169.28
23	Water conduits to and from pumping plant and reservoirs	90 710	43 796	April 30, 1930	207.12
	Drainage Pump House:				
24	Substructure by force account labor	45 382	25 088	April–September, 1931	180.89
25	Superstructure by contract	18 734	11 034	October 20, 1931	169.78
	Plant Drains and Sewers:				
26	By force account labor	60 721	33 568	April–September, 1931	180.89
27	By contract	13 365	6 527	July 10, 1929	204.77
28	Total cost of Filtration Plant (structures)	$3 298 497	$1 618 230	......	......
	(*c*) Filtration Plant Equipment				
29	Drainage pumps, electric controls, valves and piping, installed	$12 335	$7 214	September 8, 1931	$171.40

TABLE 4.—(*Continued*)

Item No.	Units constructed	Construction Cost		Date of receipt of bids	Cost index based on the year 1913 as 100
		Actual	Adjust to 1913 costs by Index Number		
	(c) Filtration Plant Equipment—(*Continued*)				
	Sluice-Gates and Valves:				
30	In coagulation basins	$53 197	$25 784	October 21, 1929 January 6, 1930	$206.32
31	In mixing chamber	135 970	65 903	October 15, 1929	206.32
32	Mechanical mixing equipment, installed (estimated)	30 000	17 647	(Future)	170
33	Raw-water meters and gauges, installed	9 326	4 697	November 11, 1930	198.54
34	Filtered water meters and gauges, installed	4 096	2 137	April, 1931	191.63
35	Water-level gauges and signal system, installed	15 545	8 303	June, 1931	187.23
36	Pressure piping in coagulation basins, installed	4 338	2 291	May, 1931	189.33
37	Stop-logs and flash-boards, installed	1 451	766	May, 1931	189.33
38	Auxiliary equipment, including pressure pumps and sump pumps in pipe galleries, installed	5 947	3 104	April 14, 1931	191.63
39	Sample pumps and piping	868	453	....	191.63
40	Wash-water pumps and piping, including meters and valves, installed	79 921	39 772	July 15, 1930	200.95
41	Chlorinating equipment and piping, including feeders, scales, and all dosing lines, installed	33 257	17 098	March 31, 1931	194.51
42	Total cost of Filtration Plant equipment, installed	$386 251	$195 169	....	....
	(d) Filter Equipment				
43	Filter operating tables, including pressure tubing, installed	$55 547	$27 952	October 7, 1930	$198.72
	Filter Rate Controllers and Gauges:				
44	Delivered	93 958	45 220	March 6, 1929	207.78
45	Installed by contract	16 377	7 998	July 10, 1929	204.77
	Filter Valves:				
46	Delivered	214 562	103 095	March 2, 1929 November 1, 1929	208.12
47	Installed by contract	5 297	2 587	July 10, 1929	204.77
	Filter Piping and Special Castings:				
48	Delivered	76 614	37 345	May 16, 1929	205.15
49	Installed by contract	10 603	5 178	July 10, 1929	204.77
	Wash-Water Troughs:				
50	Delivered	86 553	42 190	May 16, 1929	205.15
51	Installed by contract	10 773	5 261	July 10, 1929	204.77
	Filter Under-Drain Piping and Supports:				
52	Delivered	46 627	23 975	January 16, 1931	194.48
53	Installed by force account labor	8 241	4 726	July, 1931	174.37
	Filter Gravel:				
54	Delivered	17 116	9 576	May–September, 1931	178.74
55	Installed by force account labor	15 668	8 766	May–September, 1931	178.74
	Filter Sand for Twenty-Two Filters:				
56	Delivered	12 264	6 305	March 24, 1931	194.51
57	Placed by force account labor	6 891	3 855	May–September, 1931	178.74
58	Miscellaneous general expense	2 783	1 392	....	200
59	Filter sand in place for forty-six filters (estimated)	37 000	21 765	(Future)	170
60	Total cost of filter equipment, installed	$716 874	$357 186	....	....
61	Total cost of Filtration Plant construction, fully equipped	$4 401 622	$2 170 585	....	....
62	Total cost of engineering, including design, supervision, and administration	409 518	197 835	....	$207
63	Total cost of Filtration Plant (300 mgd capacity)	$4 811 140	$2 368 420	....	....
64	Cost of experimental investigations (including basin-model tests, experimental filter plant, filter under-drain, sand-washing studies, pile testing, soil testing, and roof load tests)	$80 898	$39 081	....	$207

TABLE 4—*Continued.*

Item No.	Units constructed	CONSTRUCTION COST		Date of receipt of bids	Cost index based on the year 1913 as 100
		Actual	Adjust to 1913 costs by Index Number		
	(*d*) FILTER EQUIPMENT—(*Continued*)				
65	Engineering cost (percentage of total cost)	8.51			
66	Experimental work cost in percentage of total cost	1.68			
67	Filtration Plant cost per million gallons daily capacity (exclusive of experimental work)	$16 037	$7 895		
	(*e*) OFFICE AND LABORATORY BUILDING				
68	Foundation work	$31 928	$15 997	September 2, 1930	$199.58
69	Building construction	91 370	45 781	September 2, 1930	199.58
70	Heating, ventilating, and plumbing	39 595	19 839	September 2, 1930	199 58
71	Electrical work, including clocks and gauges	19 598	9 820	September 2, 1930	199.58
72	Laboratory equipment	5 018	2 423	April 1, 1930	207 12
73	Engineering cost, including design, supervision, and administration	60 041	29 577		$203
74	Total cost of Office and Laboratory Building	$247 550	$123 437		
	(*f*) SUMMARY				
75	Reservoir (60 mg capacity)	$1 471 044	$761 853		
76	Filtration plant (300 mgd capacity)	4 811 140	2 368 420		
77	Experimental work	80 898	39 081		
78	Office and Laboratory Building	247 550	123 437		
79	Total	$6 610 632	$3 292 791		
80	Total engineering cost (included in Items Nos. 75 to 78)	$564 136			
81	Engineering cost in percentage of total cost	8.53			

COST ANALYSIS

Before the beginning of any construction work on this plant, a Cost Accounting Bureau was organized with competent clerks and bookkeepers to keep complete records of the actual cost of the various features of the work as nearly as could be obtained by independently taking the time and material used in the work and checking where possible with the contractors' records. The costs of the main units of the plant as compiled from the account ledger are given in Table 4. These costs represent not only the contract costs, but also all costs incidental to the work, including force account work done by direct labor. The engineering costs include all design work, checking, preparation of plans and specifications, consulting architects' and mechancial engineers' fees, printing of plans and specifications, supervision of construction, time-keeping, clerical work, purchasing expense, checking of shop details, inspection of materials, shop inspection, and general administration of the work. The cost of experimental work is listed separately. For ease in comparing with other projects the costs have also been reduced to the basis as of the year 1913 by using the construction cost index figures of the *Engineering News-Record.*

ACKNOWLEDGMENTS

The design and supervision of construction of the Springwells Filtration Plant was done by the Filtration Bureau of the Division of Engineering formed by the Board of Water Commissioners of Detroit for this purpose. This Division worked under the general superintendence of George H. Fenkell, M. Am. Soc. C. E., General Manager and Chief Engineer, and F. H. Stephenson, M. Am. Soc. C. E., Engineer of Water System, of the Department of Water Supply. The Filtration Bureau was in the general charge of A. B. Morrill, M. Am. Soc. C. E., Assistant Engineer (Filtration), with the writer in charge of design, and J. W. Orton, Assoc. M. Am. Soc. C. E., Assistant Civil Engineer in charge of construction. Mr. J. C. Thornton, Architect, was responsible for the architectural work, and the E. R. Little Company, Mechanical Engineers, consulted on the heating and ventilating design. Mr. E. W. Frey, Accountant, had charge of the keeping of costs and records.

CONCLUSIONS

The execution of this project has contributed to the progress of water purification practice: (1) In the development of knowledge of the flow in coagulation basins and distributing inlet details by experimental model investigation; (2) in the development of higher rates of filter washing and their control by sand expension; (3) in the conception and development of the shunt system of operating filtered-water reservoirs; and (4) in the improvement of details, such as low loss-of-head rate controllers, summation of Venturi meters, perforated pipe filter under-drains, and chemical handling equipment.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

MODIFYING THE PHYSIOGRAPHICAL BALANCE BY CONSERVATION MEASURES

Discussion

BY A. L. SONDEREGGER, M. AM. SOC. C. E.

A. L. SONDEREGGER,[56] M. AM. SOC. C. E. (by letter).[56a]—More than gratifying have been the discussions in that they have brought out many points of interest and importance in stream control and conservation. Like the subject of the paper, the discussions have covered a large range of thought and observation. Most of the discussers appreciate the necessity of a correct evaluation of the effect of modern works on natural processes and the physiographical balance. Engineering science has made great strides in the theoretical analysis of strains, stresses, and forces, and has succeeded in clothing important results of research in convenient formulas for the ready use of a learned and busy profession. On the other hand, the study of natural phenomena and a correct appraisal thereof proves to be a matter of experience rather than of learning and, as a rule, is not acquired without reprimand by failure. Hence, it would not be astonishing if subjects like the disturbance of the physiographical balance would find more appreciation with the older members of the profession than with the recent graduate who is justly proud of his newly acquired knowledge.

The importance of the water-shed cover and its effect on run-off and débris production is touched upon by several discussers. Mr. Lippincott's discussion is particularly illuminating in this respect, in that it is supported by comparative data relative to the effect of the vegetal cover in its natural condition and after one or more burnings. He concludes that "greater effort should be made than in the past to preserve the brush cover from fire." While this is a desirable objective, it must be borne in mind that the preservation of the brush cover over a long period of years promotes the accumulation of highly inflammable materials which, when set on fire, are difficult of

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NOTE.—The paper by A. L. Sonderegger, M. Am. Soc. C. E., was published in December, 1933, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1934, by Messrs. H. H. Chapman, and E. B. Debler; April, 1934, by Messrs. Frank E. Bonner, and C. S. Jarvis; May, 1934, by Messrs. W. P. Rowe, and J. C. Stevens; August, 1934, by Gerard H. Matthes, M. Am. Soc. C. E.; and September, 1934, by Messrs. J. B. Lippincott and Rhodes E. Rule.

[56] Cons. Engr., Los Angeles, Calif.

[56a] Received by the Secretary September 18, 1934.

control. Fires will continue to occur; in fact, with an ever growing population and greater facilities of access to the water-shed, the odds are in favor of an increasing fire hazard.

These considerations may have induced Mr. Rowe's point of view, namely, that of periodical controlled burning. Possibly the solution to this perplexing problem is already found in the present policy of the responsible authorities, of maintaining efficient fire-fighting organizations and completing the network of mountain roads which will permit the rapid conveyance of crews and apparatus to strategical points while fires are in their incipiency. While the number of fires may continue to increase, the probabilities are that, with improved methods and perfected organizations, damages will be confined to smaller areas. Thus, a system of rotation of burning may evolve which will eliminate major conflagrations, at the same time relieving authorities of the risks and responsibilities that attend controlled burning.

The La Crescenta-Montrose flood discussed by Messrs. Lippincott and Rowe, and referred to by others, has brought out several distinct facts bearing on the flood-control problem of Southern California: (1) That brush fires are a permanent menace at any point; (2) that run-off from tributaries may assume such proportions as to overtax existing flood channels of main streams; (3) that débris production from burnt-over areas may be of such magnitude as to be a menace to foothill areas and to the storage space of flood-control detention reservoirs; and (4) that a material increase in the peak discharge of both water and débris may result from the failure of temporary check dams.

Mr. Jarvis takes exception to the writer's statement that a permanent decrease or increase of 20% of the mean rainfall of semi-arid areas might result in far-reaching climatological or physiographic changes, while Mr. Stevens challenges the general statement that physiographical factors combine to produce a balance, even under existing conditions. The viewpoints are distinctly at variance.

It is admitted that, in the Southwest, cyclic fluctuations of seasonal rainfall in the past, which resulted in wet and dry periods of 10 to 15 yr each, seemingly had no effect on the development of the country, or on the earth's surface. As far as farm husbandry is concerned this is explained, in part, by the utilization of underground storage which, over large areas, yielded abundant supplementary supplies for the relatively short period of droughts. With a permanent reduction of 20% in the mean seasonal rainfall, the periods of relatively low water supply would necessarily be extended and periods of recharge correspondingly shortened, leaving no doubt as to the ultimate effect on the safe water yield of large areas.

The natural vegetal cover of the water-shed, as a rule, is a fair index of the mean seasonal rainfall, expressed in the prevailing species, as well as in the vigor and size of individual plants. In fact, with some experience in these matters, it is feasible to estimate with a fair degree of accuracy, the seasonal rainfall by the character of the vegetal cover. The difference in appearance of water-sheds receiving 12 in., 15 in., and 18 in. of mean seasonal rainfall is striking enough to leave no doubt as to Nature's adjustment to mean climatic conditions. Similarly, general topographic conditions must be the result of some combination of the effect of the controlling causes. If the latter undergo permanent changes, a reflection in effect is inevitable. Nature, like the "mills of the Gods", may grind slowly, yet it grinds exceedingly fine.

Mr. Stevens' standpoint is scarcely justified. Nature's adjustments must be to some mean condition which undergoes only gradual changes; otherwise, human settlement would be forced to make way to a nomadic existence. Exceptions to this rule, due to local disturbances by reason of floods, wind storms, fires, etc., no doubt take place, and may result in radical changes of physiographical features — often temporary, sometimes permanent — but they effect a relatively insignificant portion of the inhabitable areas.

Professor Chapman makes some rather startling assertions in that he claims (1) that the physiographical balance is maintained over large areas in the West by grasses and herbage; (2) that profound disturbances of the equilibrium have taken place through extensive over-grazing; (3) that over-grazing is the principal cause of modern arroyo formation and that the cycle of degradation now operating is of recent origin; and (4) that historical evidence shows that both the Gila and the Colorado were clear streams near Yuma in 1824.

It is not disputed that over-grazing within the water-shed of the Colorado River has reduced the grass carpet over large areas, thus promoting erosion and denudation. This phase has been discussed by Ralph I. Meeker,[57] Assoc. M. Am. Soc. C. E., and F. H. Olmsted,[58] M. Am. Soc. C. E. However, it is questioned whether over-grazing is a major phenomenon affecting the physiographic balance of the West. On the subject of the origin of silt the following quotation from the comprehensive study of silt production in the Colorado River water-shed is submitted:[59]

> "The aridity of the climate and the consequent lack of vegetation is one of the main causes. Because the territory is sparsely settled, relatively few climatic records have been kept, and in many parts the precipitation is not known accurately. Roughly estimated, 40% of the total area of the basin has a precipitation of less than 10 inches a year, in 50 per cent the precipitation ranges from 10 to 17 inches, and in the remaining 10 per cent it is from 17 to 25 inches and higher in the high mountains. Fully one-half of the basin is either bare or but scantily covered with desert shrubs and grasses."

Relative to the silt production of the Green River and Upper Colorado, where the major grazing areas are located, as compared with that of the Little Colorado and San Juan Rivers, data given in the report[60] reveal a much greater proportion of silt in the water of the lower tributaries which drain desert areas.

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[57] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and H. F. Blaney, Assoc. M. Am. Soc. C. E., *Technical Bulletin No. 67*, U. S. Dept. of Agriculture, pp. 7 and 19.

[58] "Gila River Flood Control", 1919, Senate Doc. No. 436, 65th Cong., 3d Session.

[59] *Technical Bulletin No. 67*, U. S. Dept. of Agriculture, p. 5.

[60] *Loc. cit.*, Tables 2 and 3.

Professor Chapman's reference to the diary of the explorer, James M. Pattie, in which the Gila and Colorado Rivers are described as clear streams, can scarcely be accepted as conclusive historical evidence that the muddy appearance of these streams is a modern phenomenon. Other historians have left records which emphasize conditions to the contrary. Quoting from F. S. Dellembaugh[61] who was a member of the Second Powell Expedition through the Colorado River Canyon, in 1872:

"In 1539, the Spanish explorer, Francisco de Ulloa, leaving from Acapulo, sailed to the head of the Gulf of California where he made this observation—'and thus sailing we always found more shallow water and the sea thick, black and very muddy'.

* * * * * * * * * *

"In 1609, Don Juan de Onate, traveling west from San Juan on the Rio Grande *via* Zuni, '10 leagues beyond Moki, they crossed a stream flowing northwesterly which was called "Colorado" from the color of its water.' This has been identified as the Little Colorado.

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"In 1826, Lieut. R. W. Hardy of the British Navy sailed up the Gulf of California and encountered 'a vein of red water' which later was proved to have been the Colorado.

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"On Powell's first trip down the Colorado, the tributary Fremont River, in Southeastern Utah, was given the name 'Dirty Devil' from the character of its water.

* * * * * * * * * *

"In 1869, Powell's party found the Little Colorado at its mouth 'small, muddy and saline'."

Lieut. R. S. Williamson[62] writes:

"The water [in the Colorado River, at Yuma, in 1853] was highly charged with fine red mud which gave it a decided red color and opacity. * * * The amount thus annually transported to the Gulf of California by this river must be very large, and very considerable additions to and alterations of its delta must result."

Last, but not least, the statement is here presented that the Grand Canyon of the Colorado, cut to a depth of 1 mile into the plateau during past ages, has been the basic cause for the degradation of the tributaries and their water-sheds during the same period, with the great Colorado Delta a silent but potent witness to a process in which the works of Man had no part.

The discussion by Mr. Bonner of the rate of débris production of representative streams, storage depletion through siltation, and annual silt production in the United States, illustrated in Tables 3, 4, and 5, is of general interest and no doubt is widely appreciated by engineers occupied with problems of water supply, conservation, and flood control. Table 3, in particular, has filled a long-felt want relative to the silt load of streams in that it covers a wide range of drainage areas, both as to size and geographical location. The assumption of a mean weight of 85 lb per cu ft for alluvial deposits furnishes a convenient justifiable ratio for the establishment of the relation between weight and volume and for comparison of data when different methods of measurement have been adopted. The compilation of records of storage reserve depletion is gratifying in that it demonstrates for a large majority of cases a remarkably low rate of depletion.

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[61] "The Romance of the Colorado River", pp. 7, 78, 120, 215, and 218.

[62] Report of Explorations in California, 1853, Senate Doc. No. 78, 33d Cong., 2d Session, p. 112.

Reference has been made to the conditions on the South Pacific slope which form rather the exception to this rule. Because of steep gradients of streams and, in many cases, unsuitable foundations, economic reservoir sites are few, and the available storage capacity, almost without exception, is deficient for flood control. Flood regulation, therefore, remains imperfect, being circumscribed by available detention storage. On many streams, the one practical reservoir site has been utilized and the channel capacity below it has been adjusted to the resulting peak flow. Although, under normal conditions of the water-shed cover, débris production may not be a disturbing factor, there remains the ever-present danger of abnormal erosion from burned areas. Under such conditions, a preconceived plan of débris storage for the protection of water storage becomes imperative.

Check Dams.—Mr. Jarvis comments on the relative merits of high débris barriers and check dams for conservation projects. Check dams were an important feature in the La Crescenta-Montrose flood discussed by Messrs. Rowe, Lippincott, and Rule.

These check dams were of a temporary type of construction. Many had been placed in the small canyons of the La Crescenta-Montrose area by the Los Angeles County Flood Control District several years prior to the fire. Immediately after the fire of November, 1933, other agencies in the vicinity were engaged to build additional checks, both in the canyons and on the débris cones. In Pickens Canyon, checks had been placed on the cone from the State Highway to the mouth of the canyon and from there in the rock formation to upper reaches of the creek. This work was of such magnitude as to be generally known and to create in the local population the conviction that the problem of flood control had been satisfactorily solved. As an added precaution, the Forestry Service had sowed a large part of the burned-over area with a variety of mustard used in Southern California for cover crops. The criticism has been advanced that a widespread use of temporary structures in the mountain areas tends to create a false sense of security on the part of the population of the area below. This unfortunate psychological effect, and the dangers inherent in temporary types of structures have also been brought out by Mr. Debler.

In this particular case, the checks undoubtedly released a considerable quantity of débris which, with permanent structures, might have remained in the mountains. However, with a subsequent flood, their efficiency would have been negligible in any case because they would have been either completely back-filled or buried. The latter effect was observed in 1924 in Sawpit Canyon where the stream bed, raised by a series of checks, assumed its original gradient.

Check dams have been an element in the flood protection and conservation program of Southern California. Thousands have been built and although those subjected to severe tests have failed, the popular faith in and demand for this type of structure has not abated.

The conception has been that check dams "hold the water in the mountains" for future release, affecting not only conservation, but also regulation of flood flow. So firmly has this idea been entrenched in the public mind that check dams have lost their technical significance and in Southern California have become a political issue. For this reason, it is considered justifiable to devote some space to an analysis of the function of this type of structure.

As a flood-control feature check dams were proposed in Southern California after the capital floods of 1914 and 1916, the idea being to regulate the flow of the mountain streams at the source. The prototype of the check-dam system was the débris barrier of the Swiss Alps. Unfortunately, in applying the Swiss system to conditions in Southern California, the true functions of the structures were misunderstood, and the guiding principles of construction disregarded. In the Alps, the barrier systems were utilized to control abornmal débris production in regions where the formation consists of soft slates, marls, and chalks, which are easily erodible. The Alpine barriers were built of massive dry masonry, as a rule, with the two top courses of pointed rock laid in cement mortar, and were from 20 to 100 ft in height. Particular care was exercised to secure the foundation by building on bed-rock, or by providing one or more stilling-basins as a protection.

The purpose of these barriers is first to stop erosion of the stream bed and then to cause the back-fill of the barrier and thereby protect the adjacent slopes from undercutting and slides. After the slopes are stabilized, they are further consolidated by planting. The massive type of dry masonry is well demonstrated in reports of Swiss engineers on the improvement of mountain streams.[63]

In the application of checks to Southern California conditions, the intention was primarily to regulate the flow of water during flood peaks and to effect conservation by holding water temporarily in the back-fill of the dams. This was to be accomplished by the erection of systems of check dams of a nominal height of 6 ft. distributed throughout the length of the stream bed. In some canyons several hundred checks were erected. Débris control, or storage, has not been considered the major objective.

The original plan contemplated the construction of dry masonry arches, built carefully to bring the arch action into play. However, lack of skilled labor, unsuitable materials, and the absence of proper foundations soon caused a modification in design. Arches were abandoned, and the dry masonry degenerated into dry rock walls, built by unskilled labor. Cases are known in which, in the absence of suitable quarry rock, boulders from the bed of the creek were used. After numerous failures, the practice of wrapping the structures with triangular wire netting was adopted, so that, to-day, the strength of the rock wall rests with the strength of the wire mesh. Large numbers of such dams have been built in Southern California. The contention has always been made that these dams can be built for from $25 to $50, although experience has shown that they cost more and that the cost of wire-bound rock check dams may be as high as $500, depending on the length. It is a feature of this check system that the dams are built approximately the same height regardless of the size of the water-shed and the depth of the stream flow, and that the spacing is more or less arbitrary. The systems, as constructed deviate further from the theoretical plan in that, in many canyons, dams cannot be built in the uppermost and steep parts where the slopes may be from 40 to 60 per cent.

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[63] "Wildbachverbauungen und Flusskorrektionen in der Schweiz", 1914, and *Proceedings*, Swiss Engrs. and Archts. Assoc. 1903.

It is apparent that the function of the barrier systems in Southern California is fundamentally different from that in the Swiss Alps and, furthermore, that the rugged construction of the latter has been abandoned for a comparatively fragile and unstable type.

Flood Regulation by Check Dams.—The popular view that "check dams hold the water in the rocky formation of mountains" may be explained by the erroneous concept that the time consumed in transit of the water from the water-shed to the valley is materially increased. It should be clear that checks do not affect the rate at which water collects from the canyon slopes so that the period of retardation is determined mainly by the amount of possible channel storage within the checked area.

Conditions are conceivable in which the construction of a proper number of correctly proportioned small checks may reduce the velocities of a mountain stream to such an extent that a temporary storage in the channel will result and, hence, a material reduction of the peak flow. A first requirement is that the height of check dams is in correct proportion to the depth of overflow (see Fig. 5(*a*)). Next, comes the spacing of check dams which is determined theoretically by the point of intersection of the original slope (20%) with the slope of the back-fill of the check (5%). In other words, the spacing must be a function (*a*) of the height of the check; (*b*) of the original gradient of the stream; and (*c*) of the gradient normally assumed by the back-fill material. This proportioning holds for a depth of overflow which will produce a free fall and a destruction of the energy by fall and turbulence. However, if the depth of overflow is such as to cause the check to act as a submerged weir, the effect of the drop is largely lost. This, for example, was the case with some of the check dams in the La Crescenta-Montrose area (see Fig. 5(*b*)).

Aside from any question of design, regulation is a function of the total reduction in velocity during peak periods. This reduction results from decrease in gradient, greater width of channel, and loss of energy at the various drops. Theoretically, the reduction of the gradient from 20% to 5% would reduce the mean velocity by about one-third; the flat section above checks tends further to retard the flow, while the increased depth of water in the narrower parts of the channel below the check increases the theoretical gradient. The resulting channel storage will be in proportion to the length of the checked section and the reduction of velocity.

Apparently, the most efficient reduction in gradient compatible with economy would result from high barriers spaced correspondingly, for the reason that the narrow section of the channel would be only a small part of the total length between checks, leaving a large stretch for the adjustment of velocities. With barriers of a height of 20 ft, or more, the width of channel at the check would probably be in excess of 50 ft. Under such conditions, channel storage would attain a maximum.

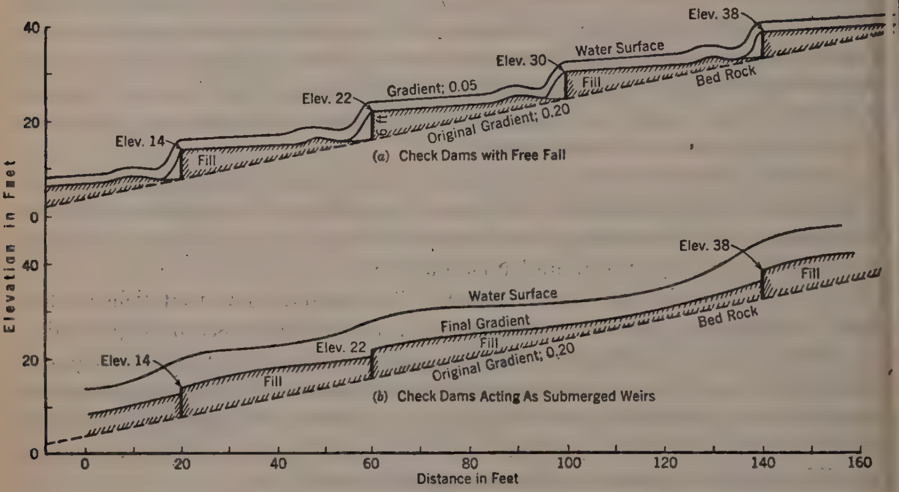

FIG. 5.

Assume the theoretical case of a canyon being visited by a rain of maximum intensity. In such an event, the hydrograph of an unchecked canyon would assume the line, *A-B*, shown in Fig. 6, with a peak discharge represented by Line *B-C* and the run out by Line *C-D*, the period of the storm being indicated by the distance, *T*. If, as a result of proper checking, peak velocities have been reduced, the time represented by Curve *A-B* becomes longer and if the peak is delayed beyond Point *C*, it will also be reduced as indicated by Curve A-B_1-C_1. In other words, factors remaining

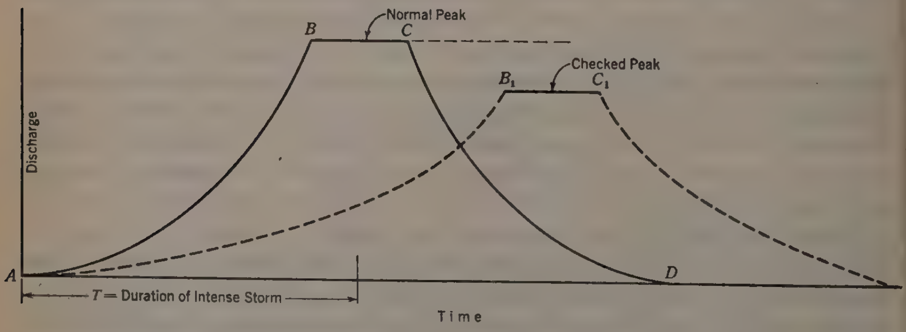

FIG. 6.

equal, the shorter the time represented by Line *T* (that is, the duration of the storm), the greater the probability that checking would be effective. On the other hand, if the duration of the storm represented by the distance, *T*, and the peak flow, *B-C*, are extended beyond B_1, checking would have no effect on the size of the peak; the effect would be a delayed peak of shorter duration. In any case, the delay could not be longer than the difference in time of transit for the water from the upper end of the canyon to the mouth.

The effect of checking can best be explained by an example. Assume a mountain stream with hydraulic properties, as given in Table 10. While the total difference in channel storage is 100 000 cu ft, the proportion of this extra storage which affects the peak flow depends on the depth of stream flow prior to the period of intense rainfall and, consequently, can be only a portion of the 100 000 cu ft. Furthermore, if the time of the peak discharge exceeds 833 sec, the maximum flow at the canyon mouth remains the same in both instances.

TABLE 10.—HYDRAULIC PROPERTIES OF AN ASSUMED MOUNTAIN STREAM

Description	Unchecked stream	Checked stream
Length of bed, in feet	5 000	5 000
Peak flow, in Cubic Feet per Second:		
Upper end	100	100
Lower end	500	500
Mean velocity, in feet per second	10	6
Mean Wetted Area, in Square Feet:		
Upper end	10	$16\frac{2}{3}$
Lower end	50	$83\frac{1}{3}$
Channel storage, in cubic feet	150 000	250 000
Time of transit, in seconds	500	833

In Southern California, the small tributary canyons, in which checks are placed, are short, ranging in length from ½ mile to 1½ miles, with gradients of from 10 to 40%; hence, velocities are high and time of transit relatively short—probably less than 30 min for severe storms. Cases are recorded where two periods of intense rainfall occurred at short intervals. Under such conditions the second peak might catch up with the delayed first peak of a checked canyon, resulting in a greater flood than would happen under natural stream flow.

The difficulty in estimating the effect of check dams lies in the lack of reliable data as to the intensity and duration of storms. It is unwise, therefore, to place too much, if any, reliance on the effective flood protection by systems of checks, even if properly designed and constructed. In fact, it is the writer's opinion that, during capital floods, conditions may be such as to render regulation by check dams ineffective.

Water Conservation by Check Dams.—In an appraisal of the efficiency of check dams for purposes of water conservation, differentiation must be made between streams flowing on bed-rock and those flowing on an alluvial fill.

Mountain streams issuing from bed-rock canyons act essentially as drains, intercepting the water which percolates down the slopes in fissures, cracks, and in the soil cover. If the creek bed is crossed by deep-seated fissures, the stream will be carrying the unabsorbed water. This function of the stream as a drain is not changed by the construction of a series of check dams, except that the débris behind the checks will hold some of the water temporarily, which then will be exposed to evaporation from the saturated mass, or will promote plant growth. Absorption into deep-seated fissures is assisted by this process only to a small extent.

A stream flowing on porous alluvium, with a water-table at effective depths, is naturally a losing stream. If checked by a system of dams, absorption will increase in proportion to the additional width of the wetted perimeter. In both cases checks should be credited with the storage in the débris which, although small, nevertheless remains an element to be considered. However, the débris storage and its water capacity increase more than the square of the height of the dam; hence, conservation favors the higher dams.

Summarizing on the relative merits of low and high check dams, it is concluded that for the consolidation of the stream bed in an erodible formation, low checks or fixed sills offer a satisfactory solution; that for débris storage, the consolidation of slopes, and, for conservation, high barriers may be found to be more efficient; and that for purposes of regulation, barriers with negligible water storage cannot be relied upon at times of critical discharge.

Unfortunately, the popular conception may attribute flood regulation to a system of checks designed for conservation, or débris control, and induce encroachment on the over-flow areas of a stream. Unless such areas are definitely zoned as dangerous for habitation, destruction of human life and property is inevitable.

Design of Check Dams.—To be effective check dams must have a height in proper relation: (*a*) To the depth of overflow; (*b*) to the gradient assumed by the stored débris; and (*c*) to the distance between checks. These matters would be taken into consideration were the problem one of designing a small number of high barriers; yet they are equally important with small checks.

Another feature of the design would be the correct thickness of the dam relative to the overflow wherein must be considered such forces or combined forces as extreme high water, débris, and mud flow, resulting in an extremely high specific gravity for the material passing over the dam and the impact of large rocks. Depending on the magnitude of slides, the stream may become temporarily blocked or dammed until a small lake is formed. When such slides are topped they may be released suddenly, advancing down the canyon as a mud-flow or débris wave. Such conditions were observed in the La Crescenta-Montrose flood. To withstand such forces in steep mountain canyons will require structures quite different from a dam consisting of loosely piled rock wrapped with triangular wire netting.

The conclusion is reached that in order to be permanent a 6-ft check dam must be constructed to withstand strains and forces out of proportion to its size, and, therefore, if properly built, will prove likely to be the most expensive type of control works instead of the cheapest, as has often been claimed.

The discussions of the paper necessarily centered on conditions in the arid Southwest. The writer is indebted to Mr. Matthes for his comparison of the problem in Eastern and Western streams. Attention is called to a paper by Mr. Matthes, which bears on certain phases of the physiographical balance manifested in stream dynamics.[64]

It was the purpose of this paper to awaken the consciousness of the Engineering Profession to the dangers accompanying the disturbance of the physiographic balance. The interest thus aroused is gratifying.

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[64] "Floods and Their Economic Importance", National Research Council, *Transactions*, Am. Geophysical Union, Pt. II, 1934, p. 427.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

AN APPROACH TO DETERMINATE STREAM FLOW

Discussion

BY R. L. GREGORY AND C. E. ARNOLD, ASSOC. MEMBERS, AM. SOC. C. E.

R. L. GREGORY[21] AND C. E. ARNOLD,[22] ASSOC. MEMBERS, AM. SOC. C. E. (by letter)[22a].—Based on the assumption that certain statements contained, and procedure outlined, in a paper[23] by L. K. Sherman, M. Am. Soc. C. E., are correct, the author has presented an ingenuous method for the determination of stream flow, which utilizes some of the factors occurring in the writers' formulas.[24] The method constitutes an approach to the construction of hydrographs at points on streams for which adequate flow records are not available. Engineers in general, and the writers in particular, fully appreciate the value of any thoughts, data, or methods that tend to advance the solution of this problem.

Previously, in his discussion of the writers' paper,[25] the author had separated and combined the run-off factors pertaining strictly to the drainage area from those influenced more directly by rainfall. This combination, Equation (1), omits the three remaining run-off factors, C, A, and R_H, contained in the general run-off equation.[26]

There can be no doubt that Equation (1) may serve as a convenient instrument for aiding in the analyzation and presentation of drainage data. The author, through its use, has already presented valuable data for consideration by the profession. There has arisen in the minds of the writers, however, a doubt as to the wisdom of using Equation (1) in a procedure

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NOTE.—The paper by Merrill M. Bernard, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. Discussion in this paper has appeared in *Proceedings*, as follows: March, 1934, by C. S. Jarvis, M. Am. Soc. C. E.; April, 1934, by LeRoy K. Sherman, M. Am. Soc. C. E.; May, 1934, by W. W. Horner, M. Am. Soc. C. E.; and September, 1934, by Messrs. C. H. Eiffert and Charles S. Bennett.

[21] Chf. Insp., County of Los Angeles. Storm Drain Div., Los Angeles, Calif.

[22] Los Angeles, Calif.

[22a] Received by the Secretary July 30, 1934.

[23] "Streamflow from Rainfall by Unit Graph Method", *Engineering News-Record*, April 7, 1932, p. 501.

[24] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1038.

[25] *Loc. cit.*, p. 1150.

[26] *Loc. cit.*, p. 1059, Equation (24).

based on hypotheses that inherently prohibit consideration of changes in the hydrograph due to differences in the magnitudes of storm intensities.

In his paper,[23] Mr. Sherman states, "for the same drainage area, however, there is a definite total flood period corresponding to a given rainfall, and all one-day rainfalls, regardless of intensity, will give the same length of base of the hydrograph"; that is, the same total period of run-off. As far as the writers have been able to determine, data have not been presented to substantiate the truth of this unqualified assertion. The statement could not be true, since a difference in the magnitude of intensity for a given time period would alter the quantity of water flowing and, consequently, the time of concentration, and the relative shape and length of the graph for the total period of run-off.

For like reasons, and others, inspection leads to the conclusion that Mr. Sherman's Fig. 1 and the equations immediately following convey erroneous conceptions of the manner in which water combines in a drainage area to cause run-off.[23] Therein exists a disregard of the fundamental principle that the velocity of flow of water in streams and channels is a function of the quantity flowing.

The author's method embodies the unit-graph procedure as exemplified in his Table 3.

It is comparatively easy to make or to deny assertions, but sometimes difficult and tedious to prove their truth or falsity. The purpose of this discussion is to test the fundamentality of the procedure that the author follows in his method and to arrive at some measure of its departure from the truth in the construction of hydrographs.

To ascertain the effect on a stream hydrograph due solely to a difference in the quantity of water running requires the complete elimination of all variables with the exception of intensity of precipitation. Due to the many and varying factors and conditions contributing to flow of a drainage area, it is almost impossible to find a single experiment wherein all variables are definitely known.

For their purpose the writers have chosen a hypothetical drainage area; and have stripped it of all pondage, channel constrictions, and other features, which tend only to confuse the issue.

Consider a triangular water-shed[27] with streams concentrating along lateral lines to the main channel of flow, to be divided, as in Fig. 7, into time-contours, all points on which are equi-distant in time from the outlet.

Assume any definite proportions for the water-shed (say, ratio of length, L, to bottom width is 5 to 3). Then (all computations by slide-rule) since $L \propto A^{\frac{1}{2}}$, or $L = KA^{\frac{1}{2}}$, $K = 380.8$, in which, A = area, in acres, and L = length, in feet.

In a formula derived by the writers,[28]

$$v = 0.03162 F^2 \; S^{\frac{3}{8}} \; Q^{\frac{1}{4}} \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

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[27] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1068, Fig. 5(*a*).

[28] *Loc. cit.*, p. 1094, Equation (50).

Let $v = K_1 \, Q^{\frac{1}{4}} = 0.3944 \, Q^{\frac{1}{4}}$ (which is the equivalent of assuming that $F = 4.02$, with $S = 0.5$, or other combinations of these factors).

It has been shown[29] that for a triangular water-shed of which Fig. 7 is a special case, subjected to a uniform intensity, the value of $P = 0.5$, $Q \propto A$, and $v \propto t$, the run-off coefficient being assumed as constant. Since $v \propto t$ and $v \propto Q^{\frac{1}{4}}$, then $t \propto Q^{\frac{1}{4}}$, or $Q \propto t^4$. To simplify computations and eliminate confusion, the run-off coefficient is herein assumed as equal to 1; that is, all rain that falls runs off.

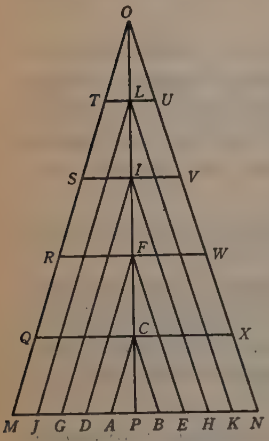

FIG. 7.—WATER-SHED.

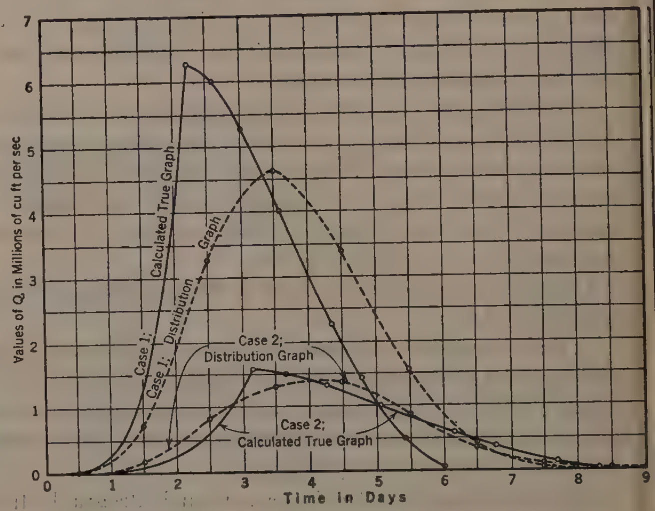

FIG. 8.—COMPARATIVE HYDROGRAPHS, CASE 1 AND CASE 2.

For a given intensity of precipitation, the water from Area *ACB*, Fig. 7, will be first to contribute its maximum rate of run-off. If it rains long enough, Areas *DFE, GIH, JLK,* and *MON,* Fig. 7, will reach their maximum contributions in sequence as time increases.

Case 1.—Assume a uniform intensity of ¼ in. per hr throughout the period of concentration of the entire area, and assign definite values to the area and its subdivisions. The hydrograph for the period of concentration is formed by Table 6(*a*), the rate, in cubic feet per second, and the time, in days, being shown in the part of the graph to the left of the peak discharge point in Fig. 8 ("Case 1, Calculated True Graph").

Case 1(a).—If the rain ceases when the full area concentrates, much water will be left on the shed in transit to the outlet. Its measure is easily found for this case. Since (as shown previously), $Q \propto t^4$,

$$Q = K_2 \, t^4 \dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

K_2 being a constant. The volume of water that has passed the outlet at the time the full area concentrates, is,

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[29] *Transactions,* Am. Soc. C. E., Vol. 96 (1932), p. 1068.

$$\int K_2\, t^4 = \frac{K_2\, t^5}{5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

in which, t = time of concentration. Since $Q_{\max}\, t = (K_2\, t^4)\, t = K_2\, t^5$ = the volume of all rain that fell, one-fifth of the water has passed the outlet and four-fifths remains on the shed. Thus, using the data opposite the area, *MON*, of Table 6(*a*), the quantity remaining on the shed, in millions of cubic feet, is:

$$\frac{4}{5}\, Q_{\max}\, t = \frac{4}{5} \times 6\,250\,000 \times 193\,200 = 966\,000$$

TABLE 6.—VALUES FOR POINTS ON HYDROGRAPH FROM BEGINNING OF RAIN TO MAXIMUM RATE OF RUN-OFF

Area (see Fig. 7) (1)	Values of area, A, in acres (2)	Length of travel, $L = 380.8\, A^{\frac{1}{2}}$, in feet (3)	Flow, Q, in cubic feet per second (4)	Velocity, $v = P\, K_1 Q^{\frac{1}{4}}$, in feet per per second (5)	TIME, $t = \frac{L}{v}$: In seconds (6)	In days (7)
			(*a*) CASE 1			
ACB	1 000 000	380 800	250 000	4.41	86 400	1
DFE	4 000 000	761 600	1 000 000	6.24	122 000	1.414
GIH	9 000 000	1 142 400	2 225 000	7.64	149 500	1.73
JLK	16 000 000	1 523 200	4 000 000	8.82	172 700	2.00
MON	25 000 000	1 904 000	6 250 000	9.86	193 200	2.236
			(*b*) CASE 2			
....	250 000	190 400	15 625	2.205	86 400	1
ACB	1 000 000	380 800	62 500	3.12	122 000	1.414
DFE	4 000 000	761 600	250 000	4.41	172 700	2
GIH	9 000 000	1 142 400	565 000	5.40	211 400	2.45
JLK	16 000 000	1 523 200	1 000 000	6.24	244 000	2.83
MON	25 000 000	1 904 000	1 562 500	6.975	272 800	3.1623

The distribution of the water remaining on the shed at the time of concentration of the entire area may be ascertained by the following procedure: Find the quantities that have passed the various time-contour lines during the time of the rain. From the total rain, falling on the area above a given time-contour during the period, deduct the quantity passed, the remainder being the quantity left on the area.

For example, consider the time-contour, *DFE* (Fig. 7). At the end of the first day of the rain, Area *GIHEFD* (A = 5 000 000 acres; Q = 1 000 000 cu ft per sec) was contributing its maximum quantity to the contour. At the end of 1.414 days of rain, Area *JLKEFD* (A = 12 000 000 acres; Q = 3 000 000 cu ft per sec) was contributing its maximum. The entire area (A = 21 000 000 acres; Q = 5 250 000 cu ft per sec) above the time-contour was contributing its maximum after 1.73 days of rain. The times of concentration, to the time-contour, of the three areas are determined from, and are the same as, those for the respective parts of the areas contributing to the main channel between *FI*, *FL*, and *FO* (see Fig. 7). In Table 7(*a*), showing times and quantities to the time of concentration to this time-contour, *DFE*, the values of Q' correspond to the time-rendering areas, the time having been calculated from Equation (9) with substitution of data opposite Area *ACB,* of Table 6(*a*) or, the time of concentration, in days, equals:

$$t = 0.04472\ (Q')^{\frac{1}{4}} \qquad (11)$$

TABLE 7.—EXPLANATORY DATA

(*a*) CASE 1			(*b*) CASE 3		
Flow, in Millions of Cubic Feet per Second		Time, in days	Flow, in Millions of Cubic Feet per Second		Time, in days
Q	*Q′*		*Q*	*Q′*	
0.44	0.04	0.633	0.21	0.01	0.4475
1.25	0.25	1.000	0.44	0.04	0.6325
3.00	1.00	1.414	0.69	0.09	0.775
5.25	2.25	1.73	0.96	0.16	0.8945
....	....	.....	1.25	0.25	1.000

In Fig. 9 the area (under the curve formed by plotting the data of Table 7(*a*)) designated *ABC,* represents the volume of water that had passed the time-contour, *DFE* (Fig. 7), to the time (1.73 days) the area above it had contributed its maximum run-off. This maximum rate of flow continues to pass the time-contour until the end of the rain, or to the time of concentration of the entire area. Hence, the total quantity of water passing the time-contour during the period of rain (2.236 days) is equal to the summation of the area (*ACB*) under the curve and the rectangle, *BCDE.* In like manner data were calculated for other time-contours and plotted in

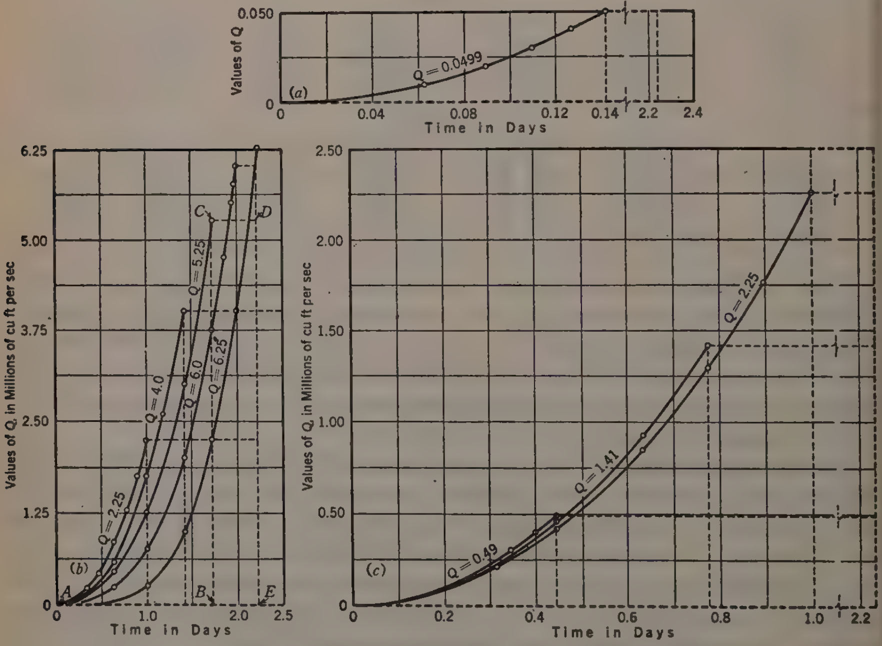

FIG. 9.—EXPLANATORY GRAPHS, CASE 1.

Fig. 9. The various contours are represented by their respective, tributary, maximum Q-values. In Table 8(a) these maximum Q-values are shown opposite the quantities, passed beyond and left on the water-shed above, the various time-contours represented.

TABLE 8.—DISTRIBUTION OF WATER LEFT ON A WATER-SHED AT THE END OF A RAIN

Flow, in millions of cubic feet per second	Rainfall and Run-Off, in Millions of Cubic Feet			Flow, in millions of cubic feet per second	Rainfall and Run-Off, in Millions of Cubic Feet		
(1)	Total rainfall (2)	Quantity passed (3)	Quantity left on water-shed (4)	(1)	Total rainfall (2)	Quantity passed (3)	Quantity left on water-shed (4)
(*a*) Case 1				(*b*) Case 2			
6.25	1 207 500	241 500	966 000	1.5625	426 900	85 400	341 500
6	1 159 000	377 000	782 000	1.5	410 000	134 400	275 600
5.25	1 016 000	446 000	570 000	1.3125	359 000	158 600	200 400
4	774 000	431 000	343 000	1.0	273 000	152 000	121 000
2.25	435 000	302 500	132 500	0.5625	153 600	106 900	46 700
1.41	273 000	209 300	63 700	0.3525	96 500	73 800	22 700
0.49	94 600	81 900	12 700	0.1225	33 500	29 000	4 500
0.0499	9 650	9 250	400	0.012475	3 410	3 260	150
(*c*) Case 3				(*d*) Case 4			
0.25	540 000	4 300	535 700	0.015625	135 000	270	134 730
0.75	518 400	18 400	500 000	0.046875	133 650	1 190	132 460
1.25	453 600	32 600	421 000	0.078125	129 600	2 090	127 510
1.75	345 600	47 100	298 500	0.140625	113 400	3 870	109 530
2.25	194 400	61 400	133 000	0.203125	86 400	5 690	80 710
1.41	121 800	58 200	63 600	0.265625	48 600	7 430	41 170
0.49	42 350	29 700	12 650	0.296875	25 650	8 400	17 250
0.0499	4 310	3 900	410	0.1225	10 580	6 100	4 480
				0.012475	1 078	937	140

When a given value of Q (Column (1), Table 8(a)) has reached the outlet, its corresponding quantity (Column (4) Table 8(a)) will remain on the water-shed. If, therefore, a quantity (Column (4), Table 8(a)) is subtracted from the total quantity left on the entire water-shed at its time of concentration (shown hereinbefore to be 966 000 millions of cubic feet), the remainder will represent the quantity that has subsequently passed the outlet when the corresponding Q-value arrives.

These quantities, together with their corresponding Q-values, were used to construct the outflow curve as plotted to the right of the peak discharge in Fig. 8 ("Calculated True Graph"). This curve of Fig. 8 represents a true hydrograph of the water-shed for the assumptions used, the area under the curve to the left of the peak discharge being one-fifth the total outflow of water and that to the right, four-fifths.

Case 2.—Assume a uniform intensity one-fourth that of Case 1, or $\frac{1}{16}$ in. per hr throughout the period of concentration of the entire area for this intensity. The hydrograph of this lighter intensity on the same area for its period of concentration is also shown in Fig. 8 ("Case 2, Calculated True Graph") as plotted from Table 6(b). Note that the time of concentration of the entire area (with the light intensity) is approximately 1.4 times as long as that for the higher one.

Case 2(a).—If the rain ceases when the full area concentrates, the same process is used in computing the data as for Case 1(*a*). The difference in results is due solely to the use of a lower intensity. Table 8(*b*) and Fig. 10 are to be compared with Table 8(*a*) and Fig. 9, respectively, of Case 1(*a*). The hydrograph is plotted in Fig. 8 and is labeled "Case 2, Calculated True Graph".

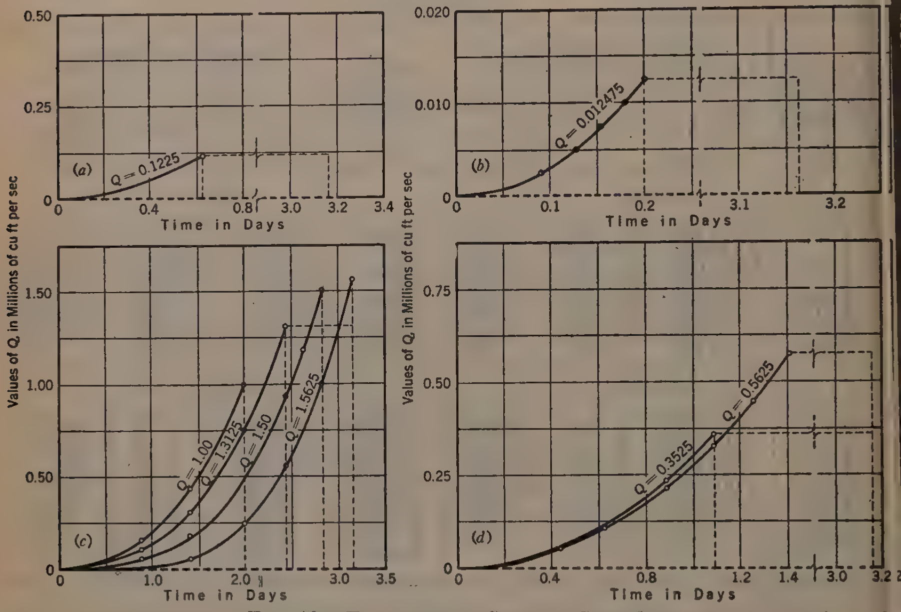

FIG. 10.—EXPLANATORY GRAPHS, CASE 2.

Equation (12) replaces Equation (11) of Case 1(*a*) since substitution of the data of Table 6(*b*) in Equation (9) renders:

$$t = 0.08944\ Q^{\frac{1}{2}} \dots\dots (12)$$

Case 3.—One-Day Duration of Rain; All Other Assumptions Identical with Case 1.—The procedure used in the solution for this case differs sufficiently from that for Case 1 to warrant a brief explanation. Note (see Table 6(*a*) and Fig. 7) that the area, ACB = 1 000 000 acres, concentrates to a maximum rate of run-off of 250 000 cu ft per sec at the end of the first day of rain. During this period one-fifth the water that fell on this area had passed the outlet and four-fifths was left on the water-shed at the end of the day—from Equations (9) and (10). Thus, the total rainfall on Area $ACB = Q\ t$ = 250 000 × 86 400 = 21 600 millions of cubic feet, of which 4 300 and 17 300 millions had passed the outlet and remained on the water-shed, respectively; but, the total rainfall on the entire water-shed during the 1-day period (see Table 6(*a*)) $= \left(\frac{A}{4}\right) t$ = 6 250 000 × 86 400 = 540 000 millions of cubic feet. Therefore, the quantity remaining on the entire water-shed at the end of the first day of rain = 540 000 − 4 320 = 535 680 millions of cubic feet.

At the instant the rainfall ceases, the same rate of run-off, 250 000 cu ft per sec, will prevail at all points between *L* and *P* of Fig. 7. At this instant the maximum *Q*-values tributary to the time-contours, *ACB, DFE, GIH,* and *JLK,* are due to contributions from Areas *DACBEF, GDFEHI, JGIHKL,* and *MJLKNO,* respectively.

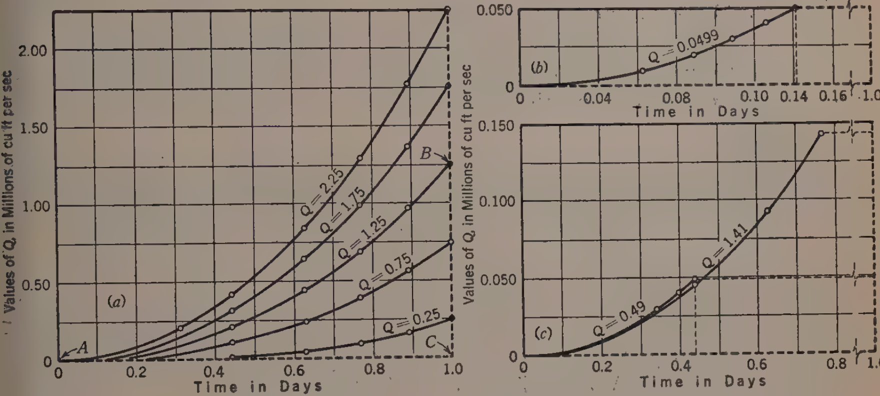

FIG. 11.—EXPLANATORY GRAPHS, CASE 3.

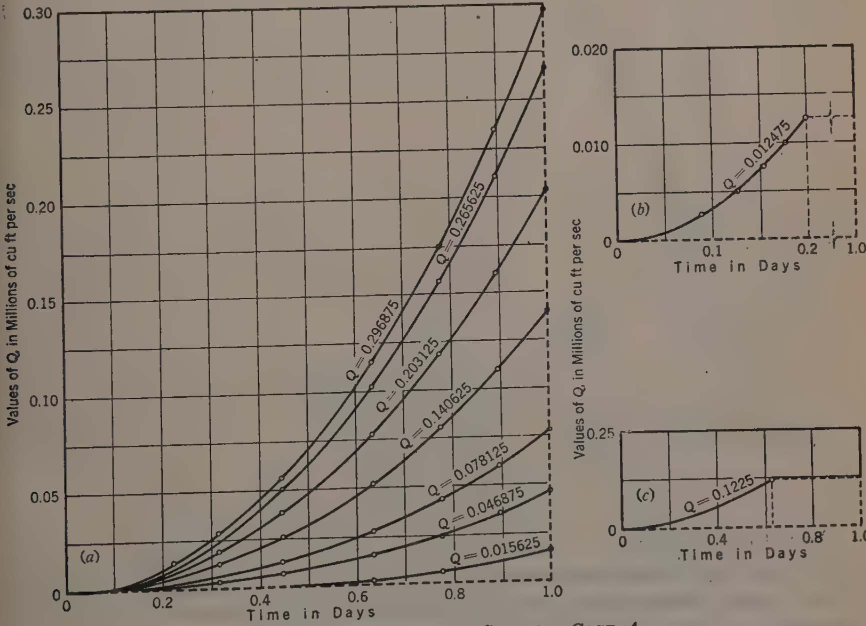

FIG. 12.—EXPLANATORY GRAPHS, CASE 4.

The quantity of water that has passed a given time-contour during the 1-day rain may be ascertained as illustrated by the example, which follows: Consider the time-contour, *DFE* (Fig. 7). Divide the tributary area

(5 000 000 acres) into sub-contours, and determine the respective values of Q, Q', and t as illustrated under Case 1(*a*). As before, the time is found from Equation (11). These values are given in Table 7(*b*) and are plotted in Fig. 11(*a*) (see Curve *AB*).

In Fig. 11(*a*) the area, *ABC*, under Curve *AB*, represents the quantity of water having passed the time-contour, *DFE*, during the day's rain. If this is deducted from the total rainfall on the water-shed above the contour, the result will be the quantity left on the water-shed at the end of the period.

In Table 8(*c*) values are given for the total rainfalls on the area above, the quantities passed, and those left on the water-shed above, the various time-contours as represented by their respective maximum Q-values.

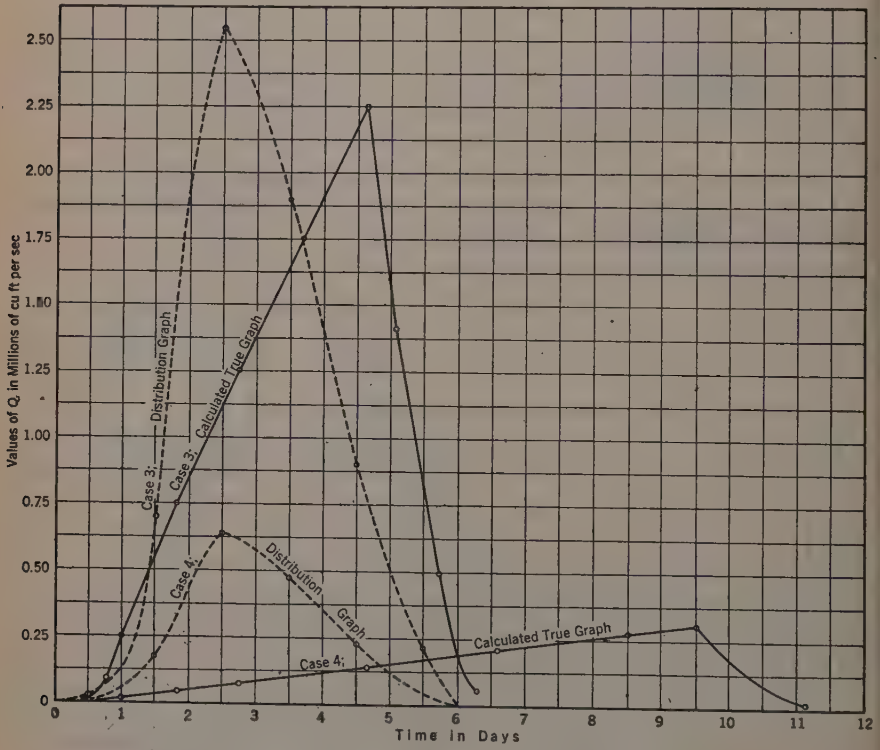

FIG. 13.—COMPARATIVE HYDROGRAPHS, CASE 3 AND CASE 4.

As before, a given quantity (Column (4), Table 8(*c*)) will still be on the shed when its corresponding Q-value (Column (1), Table 8(*c*)) arrives at the outlet. Hence, if a given quantity of Column (4) is deducted from the total quantity of water on the water-shed at the end of the rain (shown hereinbefore to be 535 700 millions of cubic feet), the result will be the quantity that has subsequently passed the outlet when the corresponding flow, Q (Column (1), Table 8(*c*)), arrives.

These quantities, with their corresponding Q-values, are plotted in Fig. 12 (see Case 3, "Calculated True Graph") which represents a true hydrograph for the assumptions of Case 3.

Case 4.—One-Day Duration of Rain; All Other Assumptions Identical with Case 2.—Following the procedure outlined for Case 3, Table 8(*d*) and Fig. 13 were constructed, due consideration being given to the effects of the lower intensity. These data are to be compared with Table 8(*c*) and Fig. 11 of Case 3, respectively. The hydrograph (see Case 4, "Calculated True Graph") platted in Fig. 12, is to be compared with that for Case 3, Fig. 12.

Comparisons.—The hydrograph (see "Calculated True Graph"), for Cases 3 and 4, may be compared directly through a study of Fig. 12. The two graphs differ radically both in length and in shape. They may be used to construct graphs following Mr. Sherman's procedure as outlined for Fig. 1 of the "Unit-Graph Method"[23]. Marked differences in results will be noted, for the two cases, as to total length of graphs, times to maximum rate of run-off, and shapes of graphs. If these graphs, so constructed, are compared with the two hydrographs of Fig. 8 (see "Calculated True Graphs"), a more complete idea as to the magnitude of errors resulting from disregard of fundamentals may be gained. Conversions to the unit-graph method are omitted herein in order to conserve space.

The distribution graph method, as proposed by the author, is more pertinent to this discussion; and for the purpose of making comparisons, data for constructing a distribution graph were selected from the hydrograph of Fig. 8 (see Case 1, "Calculated True Graph"). These data are shown in Table 9.

TABLE 9.—DISTRIBUTION GRAPH FOR CASE 1, FIG. 8

(Total Discharge = 1 207 500 Millions of Cubic Feet)

Day	Discharge, in millions of cubic feet	Approximate percentage of total discharge (distribution graph)	Day	Discharge, in millions of cubic feet	Approximate percentage of total discharge (distribution graph)
1	4 300	0.4	4	364 500	30.2
2	133 500	11.1	5	173 000	14.3
3	492 000	40.7	6	39 800	3.3

The distribution graph of Table 9 was then applied to Cases 1, 2, 3, and 4, using the data for rainfall assumed herein and following the author's method, as exemplified in his Table 3. The results are shown in Table 10 and the reproduced hydrographs are plotted in Figs. 8 and 12, as represented by the dashed curves. Little comment is necessary. Comparisons may be made directly with the calculated true hydrographs as platted on the same diagrams. In each case the variation is due to solely to the complete disregard (which is inherent in the method) of the effects of a difference in intensity.

TABLE 10.—APPLICATION OF DISTRIBUTION GRAPHS TO THE WATER-SHEDS FOR CASES 1, 2, 3, AND 4

Day No.	Daily rain-fall, in inches	GRAPH DATA Day:* 1 2 3 4 5 6 Effective percentage:* 0.4 11.1 40.7 30.2 14.3 3.3				Effective rain-fall depth, in inches	Computed flow, Q, in millions of cubic feet per second	Daily rain-fall, in inches	Graph data	Effective rain-fall depth, in inches	Computed flow, Q, in millions of cubic feet per second
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
			(a) CASE 1						(c) CASE 3		
1	6.000	0.024			.....	0.024	0.025	6.00	0.024	0.024	0.025
2	6.000	0.666	0.024		.....	0.690	0.720	.....	0.666	0.666	0.695
3	1.417	2.442	0.666	0.006	.....	3.114	3.250	.....	2.442	2.442	2.546
4	.....	1.812	2.442	0.157	.....	4.411	4.600	.....	1.812	1.812	1.890
5	.....	0.858	1.812	0.577	.....	3.247	3.390	.....	0.858	0.858	0.895
6	.....	0.198	0.858	0.428	.....	1.484	1.546	.....	0.198	0.198	0.204
7	.....	.....	0.198	0.202	.....	0.400	0.417	.....	.....	.....	.....
8	.....	.....	.....	0.047	.....	0.047	0.049	.....	.....	.....	.....
			(b) CASE 2						(n) CASE 4		
1	1.500	0.006				0.006	0.006	1.500	0.006	0.006	0.006
2	1.500	0.167	0.006			0.173	0.180	.....	0.167	0.167	0.174
3	1.500	0.611	0.167	0.006		0.784	0.816	.....	0.611	0.610	0.636
4	0.244	0.453	0.611	0.167	0.001	1.231	1.284	.....	0.453	0.453	0.472
5	.....	0.214	0.453	0.611	0.027	1.304	1.360	.....	0.214	0.214	0.225
6	.....	0.050	0.214	0.453	0.099	0.815	0.850	.....	0.050	0.050	0.052
7	.....	.....	0.050	0.214	0.074	0.338	0.352	.....	.....	.....	.....
8	.....	.....	.....	0.050	0.035	0.085	0.089	.....	.....	.....	.....
9	.....	.....	.....	.....	0.008	0.008	0.008	.....	.....	.....	.....

* Derived from Fig. 8 (see, Case 1, "Calculated True Graph.")

The variation incidental to the choice of the time-unit may be obviated by plotting the ordinates of the calculated true graphs in terms of the average flow, *Q*, per day. The resulting curves (plotted by the writers, but not published herein) still show marked variations from the distribution hydrographs.

Conclusion (a).—The water-shed selected, its size, shape, and manner of concentration, together with assumptions for intensity and characteristics, which rendered the foregoing variations, was chosen merely for the purpose of arriving at definite results for use in making comparisons. Another water-shed of any size and with other uniform intensities of the same ratio, but with the same shape, manner of concentration, and remaining assumptions, will render similar variations for comparison. Any given water-shed, similarly subjected to two different uniform intensities, will show results varying in the same general direction. Pondage, channel constrictions, and the existence of water in the channels immediately before the rains would tend to "iron out" the peaks and generally to make the variations less marked. On the other hand, perviousness of the area, causing a greater difference in the ratio of resulting run-offs than that of applied intensities, would increase the variations above those obtained for the completely impervious area assumed for the foregoing cases.

Any method or system, proposed for devising hydrographs, should at least embody within its scope the opportunity for designers to use their judgment in allowing for the effects on run-off of all fundamental factors and relations.

Conclusion (b).—The procedure, outlined and followed in the foregoing computations for the calculated true graphs of Figs. 8 and 12, although utilized for special cases, for the purpose of obtaining results to be used in making comparisons with those of the author's method, may serve as a guide in the determination of hydrographs for points on streams where adequate flow records are not available. A few more comments may serve further to clarify the applicability of this procedure to any water-shed.

Usually, it will be desired in such cases to consider a rain that lasts as long as the time of concentration of the area. The time of concentration to the maximum rate of run-off and the resulting average intensity for the assumptions may be ascertained through use of the writers' formulas and methods.[24]

Any alterations of these times and intensities, due to pondage or other delays, will depend, of course, on the judgment of the designer, as guided by any information that he may possess. These formulas and methods will also be of service in dividing the water-shed into time-contours. The distribution of the water left on the shed at the end of the rain (being the total estimated run-off less the area under the hydrograph from the beginning to the end of the rain) may be determined through the procedure outlined under Case 1.

The ordinary water-shed, of course, is irregular in shape and manner of concentration, as are also the time-contours by which it may be divided. Instead of a simple equation (as Equation (11), for Case 1) for use in conjunction with time-rendering areas in determining the quantities that have passed a given time-contour, resort should be made to the special tables[30] of the factor P, previously introduced by the writers. Since $P = \frac{v}{V}$, and since the value of V can be computed for the known quantity of water and average slope, the average velocity, v, is readily obtained. The known length divided by v will render the required time.

The detail required for a given problem and any abridgment of the method may well be left to the judgment of the engineer.

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[30] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1048, Tables 3 and 4.

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DISCUSSIONS

LOSS OF HEAD IN ACTIVATED SLUDGE AERATION CHANNELS

Discussion

BY DARWIN WADSWORTH TOWNSEND, M. AM. SOC. C. E.

DARWIN WADSWORTH TOWNSEND,[9] M. AM. SOC. C. E. (by letter).[9a]—Contributions to the subject-matter of his paper by Messrs. Thackwell, King, and Klegerman are gratefully acknowledged by the writer. Commenting generally upon these discussions, attention might well be directed again to the velocity-retarding effect attributable to air-bubble induction and the subsequent coalescence encountered in practice, as differentiated from the assumed hypothesis based upon laboratory observations made in connection with air bubbles rising through clear and quiescent water contained in glass cylinders.

That reduced cross-sectional area obtains due to the continuous presence of diffused air throughout the liquid medium, there can be little doubt. There also can be little doubt that the continuous and irregular swirling of an aerated mass results in cross-currents, up-and-down currents, and diametrically opposed currents, which would vary, continuously, the degree and character of the flow in a cross-section. For this obvious reason the writer elected not to attempt to state increased frictional resistance in terms based upon reduced cross-section and consequent increased velocity. In the writer's judgment a roughness or retarding factor, most prominent in character, is present in the flowing liquid mass.

The air bubbles contained in the aerated mass do not merely float along in the stream current in an undisturbed state as a log of wood might be expected to float along on the surface of a stream; they are decidedly and irregularly active in their continuous ascent, through expansion and coalescence, and the combination of both. If small logs instead of air bubbles were liberated in a continuous stream from a channel bottom, the flow-retarding effect would be quite dissimilar to that which might be expected from the same logs floating horizontally in the current.

NOTE.—The paper by Darwin Wadsworth Townsend, M. Am. Soc. C. E., was published in January, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1934, by H. L. Thackwell, M. Am. Soc. C .E.; and May, 1934, by Messrs. Henry R. King, and M. H. Klegerman.

[9] Cons. Engr. (Consoer, Townsend, Older & Quinlan, Inc.); Cons. Engr., Milwaukee and Met. Sewerage Comms., Milwaukee, Wis.

[9a] Received by the Secretary September 29, 1934.

It is true that the friction loss per unit of aerated channel length may be relatively small under conditions of low velocity. These losses on the other hand may be relatively large under conditions of high velocities. Furthermore, the accumulation of many relatively small friction loss increments may, and do, represent a substantial percentage of the total friction head in the design of a sizable activated sludge plant.

The conditions that obtain in a flowing aerated stream do not lend themselves ideally to head-recording accuracy. This fact the writer attempted to make clear in his discussion relative to pulsations; the results he obtained, and those encountered in practice, are sufficiently accurate to warrant their subsequent use with safety in problems of design.

It is obvious why the n-value increases as flow velocity decreases. If the upward or vertical velocity of the air-bubble stream, for instance, were greater than that of the horizontal liquid flow velocity, a curve indicating the degree of resistance would more nearly approach the vertical than would be the case if the velocities were applied in the reverse order. In other words, the same general theory of head loss and velocity resultant which applies to two horizontally flowing streams uniting with each other at an angle, applies in a somewhat modified degree in the case of the problem under discussion. In both cases it becomes necessary to build up sufficient head to overcome the resistance encountered. The trend curve (Fig. 5), which was based somewhat upon chance projection is the best illustration of the relationship between n-values and velocities.

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DISCUSSIONS

INVESTIGATION OF WEB BUCKLING IN STEEL BEAMS

Discussion

By Inge Lyse, M. Am. Soc. C. E., and H. J. Godfrey, Esq.

Inge Lyse,[8] M. Am. Soc. C. E., and H. J. Godfrey,[9] Esq. (by letter).[9a]—The data on aluminum alloy beams presented by Messrs. Moore and Hartmann are valuable additions to the results obtained on steel beams by the writers. Unfortunately, the data on the aluminum beams give only the ultimate load with no indication of the load at which the beam would become useless due to excessive deflections. As already stated in the paper, the load at failure has no significance except that it is a measure of the toughness of a beam after it has passed its usefulness. In structural design, the load at which the deflection exceeds the maximum permitted by the usefulness of the building must be taken as the basis for estimating the factor of safety. Since all the aluminum alloy beams had depth-thickness ratios of only 25 and less, buckling of the web would not take place at stresses less than the yield-point stress of the material.

No information is given as to the stress in the web at the yielding of the beam and, consequently, Messrs. Moore and Hartmann give little support to their criticism of the writers' recommendation that the average web shear be computed on the net area ($h\ t$) rather than on the gross area ($D\ t$). The observed stresses presented in Fig. 24(b) do not necessarily represent the stresses that cause yielding of the beam. H. M. Westergaard, M. Am. Soc. C. E.,[10] has shown that the concentration of shearing stress at the fillet between the web and the flange of the beam may become significant for beams having relatively small fillets. Fig. 21 also shows that the computed stresses at the yielding of the beam, instead of being greater (as they should be according to Messrs. Moore and Hartmann) are less than the yield-point stresses of the web material.

Note.—The paper by Inge Lyse, M. Am. Soc. C. E., and H. J. Godfrey, Esq., was published in February, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1934, by R. L. Moore, Esq., and E. C. Hartmann, Jun. Am. Soc. C. E.

[8] Research Associate Prof. of Eng. Materials in Chg., Fritz Eng. Laboratory, Lehigh Univ., Bethlehem, Pa.

[9] Former Research Fellow in Civ. Eng., Lehigh Univ., Bethlehem, Pa

[9a] Received by the Secretary June 18, 1934.

[10] Publication pending.

One-half the beams reported by Messrs. Moore and Hartmann failed by what has been termed web crippling. While web buckling is a failure produced by lateral instability of the web as a whole, the web crippling is a local failure produced by excessive compressive stresses. An investigation of web crippling was also conducted at Lehigh University with the co-operation of the Bethlehem Steel Company and its subsidiary the McClintic-Marshall Corporation, at the time of the tests of web buckling. The results were not included in the paper, in order to make a definite distinction between the two types of stress condition. They are presented herewith.

Web crippling is principally a local failure produced by excessive compressive stresses, and the problem of preventing its occurrence is most common in the design of structural beams supported on seat angles. The aim of the investigation was to establish the stress at which yielding due to web crippling took place.

The program included the testing of six 22-in., 58-lb, rolled, Bethlehem beams, four of which were cut from the same section and two from another section. The information for these beams which had an $\frac{h}{t}$ ratio of about 52, is given in Table 3. Three of the beams were designed to fail at the point of application of the load and the other three, at the supports. The nominal bearing lengths were 7 and 11 in. for the center failures, and $3\frac{1}{2}$ and $5\frac{1}{2}$ in. for the end failures.

TABLE 3.—WEB CRIPPLING OF BEAMS WITH $\frac{h}{t} = 52$

Beam	Web thickness, in inches, t.	Bearing length, in inches, $A + 2N$	Bearing area, in square inches	Yield-point load, in pounds	Compressive yield-point stress, in pounds per square inch	Tensile yield-point stress of material, in pounds per square inch	Ultimate load, in pounds
CT-1.....	0.397	4.74	1.88	190 000	50 500	49 000	202 000
CT-2.....	0.397	8.24	3.27	160 000	49 000	49 000	223 000
CT-3.....	0.397	6.74	2.68	220 000*	41 000*	50 000	231 500*
CT-4.....	0.397	12.24	4.86	210 000*	43 200*	50 000	264 400*
CT-5.....	0.407	4.69	1.91	160 000	41 900	44 660	205 000
CT-6.....	0.407	8.17	3.33	150 000	45 000	44 660	209 500

* Doubtful, due to thin bearing plates.

In order to prevent failure due to end twisting, steel plates were welded to the bottom flange at each end of the beam. In each end plate, at a height even with the top flange, there was a slot which served as a guide for bars welded to the center of the top flange. This arrangement, as shown in Fig. 26, allowed the top flange to deflect vertically, but restrained it from moving laterally.

All the beams were supported by rollers and loaded at the center of the span. A roller was also used for the application of the load for Beams Nos. 1 and 2, but a spherical bearing block was used for this purpose in the testing of the remaining four beams. Steel bearing plates of the proper length were used to transfer the load from the rollers and bearing block to the beam.

Vertical deflections and strain-gauge observations were taken on all beams. Except for Beam No. 1, all beams were whitewashed before testing.

Strain lines in the web appeared at both supports at a load of 115 000 lb, and, at the center, at a load of 200 000 lb in Beam No. 1. At a load of 190 000 lb, the strains increased greatly, indicating that the actual yielding of the beam occurred. The strains observed at a point immediately above the flange indicate that the yield point of the material in the web was reached at considerably lower loads. However, this local yielding did not have any effect on the beam as a whole, which continued to take load until a maximum of 202 000 lb was reached, at which time the web crippled at the end, as shown in Fig. 27.

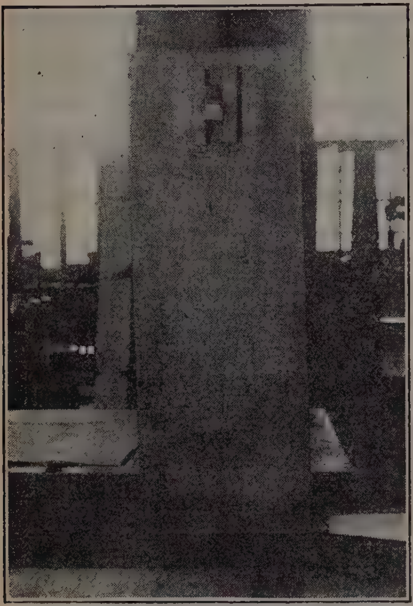

FIG. 26.—BEAM *CT*-5, SHOWING END PLATE FOR PREVENTION OF END TWISTING.

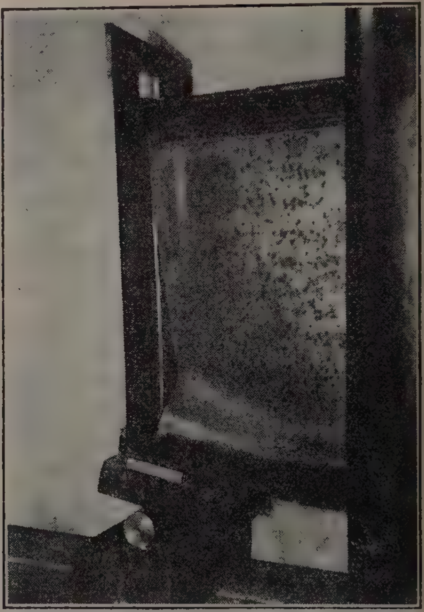

FIG. 27.—BEAM *CT*-1, SHOWING FAILURE AT THE SUPPORT DUE TO WEB CRIPPLING.

In computing the actual bearing length at the root of the web it was assumed that the stress was distributed through the flange on an angle of 45 degrees. The total bearing length thus equals the length of the bearing plate plus twice the thickness of the flange at the root of the web. The thickness of the flange was measured by micrometers at a point next to the fillet, as indicated in Table 3. The length of the bearing plate was $3\frac{1}{2}$ in. at the supports where the beam was designed to fail. The compressive stress at the root of the web at the yield point of the beam was 50 500 lb per sq in., which compares very well with the average tensile yield-point stress of 49 000 lb per sq in. for the web material. The full yield-point stress of the web was thus utilized in this beam.

Beam No. 2 was designed to fail at the loading point where the length of the bearing plate was 7 in. Strain lines appeared at the supports at a load of 100 000 lb and at the loading point at 140 000 lb. Since the failure of this beam occurred at the loading point no definite indication of yielding could be obtained from the deflection curves. The strain-gauge results showed that the strains near the root of the web increased at a high rate above a load of 120 000 lb. The strains measured at the center of the web indicated a yielding at a load of 160 000 lb, which was taken as the yield-point load of the beam. The beam continued to take load until a maximum of 223 000 lb was reached, at which time the web crippled at the loading point.

At the yield point of the beam the maximum compressive stress at the root of the web was 49 000 lb per sq in., or the same as the tensile yield-point stress of the web material.

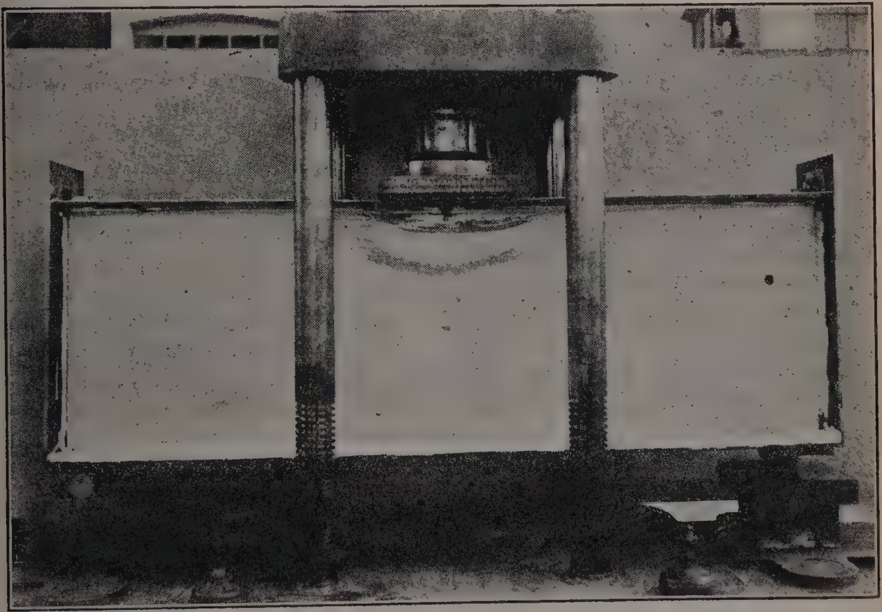

FIG. 28.—BEAM *CT*-4, SHOWING FAILURE AT THE CENTER DUE TO WEB CRIPPLING.

Beam No. 3 was designed to fail at the supports where the length of the bearing plates was $5\frac{1}{2}$ in. To prevent failure at the center, stiffeners were welded to both sides of the web at this point.

The first strain lines appeared in the web near the center at a load of 60 000 lb. The strain lines continued to form and, at a load of 110 000 lb, they appeared in the web at the supports. The observed strains indicated yielding at a load of 220 000 lb, which was taken as the yield point of the beam, while the deflection curves indicated yielding at a load of 230 000 lb. At the maximum load of 231 500 lb, the web crippled at one of the supports. The bearing plate used for this beam was too thin for effective distribution of the load from the roller to the flange of the beam and, consequently, contributed to a concentration of stress at the center of the plate, causing local failure at low average stress.

The maximum compressive stress at the root of the web was 41 000 lb per sq in. at the yield point of the beam. This is considerably less than the tensile yield-point stress of 50 000 lb per sq in. for the material in the web.

As Beam No. 4 was designed to fail at the center, stiffeners were welded to the web at the supports. The length of the bearing plate at the center of this beam was 11 in.

The first strain lines appeared in the web at the supports at a load of 90 000 lb and, at 120 000 lb, they appeared under the loading point. The observed strains indicated a yielding at a load of 210 000 lb which was taken as the yield-point load. The beam continued to take load until a maximum of 264 400 lb was reached, at which time the web crippled at the center of the beam, as shown in Fig. 28. Also, for this beam, the bearing plate was too thin for uniform distribution of the load.

At the yield-point load the maximum compressive stress at the root of the web was 43 200 lb per sq in., which is considerably less than the tensile yield-point stress of the web material.

Beam No. 5 was similar to Beam No. 1, having a bearing length of 3½ in. at the supports where it was designed to fail. The first strain lines appeared in the web at the supports at a load of 60 000 lb, and at the loading point at a load of 130 000 lb. The vertical deflections, as well as the strains, showed that the beam yielded at a load of 160 000 lb. A slight crippling of the web at the support was noted at 190 000 lb, and, at a load of 205 000 lb, the web crippled.

At the yield-point load of 160 000 lb, the computed stress at the root of the web was 41 900 lb per sq in., which compares favorably with the yield-point stress of 44 660 lb per sq in. for the web material.

Beam No. 6 was similar to Beam No. 2, having a bearing length of 7 in. at the loading point. As this beam failed at the center, the vertical deflections indicated that there was no yielding of the beam as a whole. The strain curves, however, indicated a bending tendency in the web at a load of 80 000 lb. The rate of strains increased sharply at a load of 150 000 lb, which was taken as the yield point of the beam. The beam continued to take load until a maximum of 209 500 lb was reached and, at this load, the web crippled under the center.

At the yield-point load of the beam the maximum compressive stress at the root of the web was 45 000 lb per sq in., which is close to the yield-point stress of 44 660 lb per sq in. for the web material.

Both the flexural and shearing stresses were low in all these beams, so there was no indication of yielding due to these stresses.

Since only six beams were tested in this investigation, no general conclusions can be drawn. However, the results indicate that:

1.—The appearance of the first strain lines has no relation to the yielding of the beam as a whole, but is an indication of high local stress.

2.—The compressive stress at the root of the web at the yield point of the beam corresponds quite well with the tensile yield-point stress of the material for properly designed bearing plates.

3.—The yielding was caused by the compressive stresses at the root of the web.

4.—The following formula may be used in computing the compressive stress at the root of the web: $f_c = \dfrac{R}{t(A + 2\ N)}$, in which, f = compressive stress; R = load on bearing plate; t = thickness of web; A = length of bearing plate; and N = thickness of flange at edge of fillet.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ANALYSIS OF SHEET-PILE BULKHEADS

Discussion

BY DR. ING. E. h. O. FRANZIUS

DR. ING. E. h. O. FRANZIUS[80] (by letter).[80a]—That present design methods are incomplete has been correctly emphasized in this paper. Time and time again even engineers of note resort to irrational methods as soon as they encounter difficulties in the design of bulkheads. One of the most favored "tricks" has been the assumption of restraint at the bottom of the sheet-piling, without an analysis of the forces producing such restraint. As long as there is no proof that such forces occur, the introduction of a restraint is equivalent to the appearance of a *Deus ex machina,* who is expected to deliver the designer from embarrassment. The fact that bulkheads designed in this manner have not failed is not due to the quality of the design, but to the fact that the active forces are smaller than assumed, owing to cohesion.

An attempt is made in the paper to determine the forces that act on a sheet-pile wall from the observed deflections and, finally, a new method of analysis is presented. This method of analysis is an ingenious, progressive step, and will materially advance designers in this field of engineering in their struggle for cognition.

Evidently, the principal objective of the tests was to observe the elastic line of a sheet-pile wall under load and to determine from this line the forces causing it. As it was not possible to measure the earth pressures directly, it was necessary to determine them through calculation in such a way as to satisfy the observed deflections. In doing so two errors did occur which are of decisive importance in the evaluation of these forces:

1.—In computing the active earth pressure in the tank by means of Coulomb's formula, the author neglects the friction on the side walls. The writer's experiments with large earth-pressure testing apparatus in the Laboratory for Foundation and Hydraulic Tests of the Institute of Technology, of

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NOTE.—The paper by Paul Baumann, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by Jacob Feld, M. Am. Soc. C. E.; August, 1934, by Messrs. R. L. Vaughn, M. A. Drucker, and Raymond P. Pennoyer; and October, 1934, by D. P. Krynine, M. Am. Soc. C. E.

[80] Prof., Inst. of Technology, Hannover, Germany.

[80a] Received by the Secretary, October 8, 1934.

Hannover, Germany, showed this side-wall friction to be of decisive importance. In some cases the active earth pressure was reduced to one-half, due to this side-wall friction. In fact, the latter is equivalent to a lateral anchorage of the sliding wedge. Furthermore, the distance between the front and the back wall of the test tank is too small for the active sliding wedge to develop fully. Both these effects result in a reduction of the active pressure. On the other hand, the sand contains some silt which tends to increase the active pressure beyond the value obtained by Coulomb's formula. It is not possible, therefore, to determine the effective, actual pressure without direct and exact test.

In his tests with two large testing apparatus, the writer found it necessary to lubricate the side walls with soap paste after first covering them with several layers of paper. The side-wall friction was reduced thereby to about 10% of its original value so as to render its effect on the total earth pressure quite small.

A further uncertainty is inherent in the angle of internal friction of soil under water. The author conducted three tests to determine this angle, two of which gave 31° and the third, 30 degrees. As the latter test called for the greatest movement of sand particles, prior to finding their equilibrium position, the writer would consider the greatest angle (that is, 31°) as closer to the true one. However, the difference is too small to be of decisive influence, although combined with other values it may assume a certain significance.

That the angle of 31° for the material under water is much smaller than the angle for the material "in the dry" is due to silt in the sand. The writer's tests showed that the angle of friction of clean sand under water is exactly the same as in dry sand.

2.—If these uncertainties affecting the active pressure were of slight importance the actual forces due to passive resistance in front of the wall could be found with fair approximation. The earth-resistance values obtained by the writer in his tests at Hannover (and which the author uses as a basis of his derivations) are due to a wall width of 2 m (6.56 ft), which means that the resistance values found to establish equilibrium according to the author's new method of analysis would be about four times as great. The values as measured by the writer are for clean, granular sand with practically no cohesion. On the other hand, the soil tested by the author had ingredients which, conditions permitting, will produce cohesion, although with the material under water the cohesion would be almost negligible. However, as the soil used in the author's tests has constants that differ from that used by the writer, an immediate comparison is impossible. One conclusion is certain, however, the earth resistance values as determined by the author are not due entirely to the prism directly in front of the wall; they are also due to the wedges that fan out laterally beyond a line through the side walls. The writer has had the influence of these side wedges investigated through tests on anchor-plates.[31] The top of these anchor-plater was some distance below the ground surface. The tests showed that the passive resistance was more than twice the value reported by Krey and more than three times that based on the Coulomb theory.

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[31] "Erdwiderstand auf Ankerplatten", von Dr. Ing. Wilhelm Buchholz, Pre-Print from the Yearbook of the Hafenbautechnische Gesellschaft, Vol. 12, 1930–31.

The same must be true of the author's tests. The influence of side friction is great. If the test wall had been twice or three times as wide as it actually was, the values of passive resistance would have been quite different because of the reduction in the influence of side friction to one-half and one-third, respectively. The relative magnitude of side friction and passive resistance can only be determined through tests with specially constructed apparatus.

It is much to the writer's regret to conclude that the passive resistance values, as established by the author's tests, do not truly represent the passive resistance value of that soil per unit width, of a wall of infinite width. This is also reflected in the permanent deformation of the sheet-piles (Fig. 6), as the outside piles show smaller deflections than the inside ones, due to the influence of the lateral fanning out of the resisting wedge.

As the deflections below the ground surface could not be measured, a considerable part of the elastic line and, in fact, the part subjected to the greatest loads, was not established. The results to date merit an extension of these tests by the installation of such apparatus in laboratories as will permit measuring the passive resistance of soil directly.

Naturally, the new method of analysis proposed in the paper is entirely independent of the magnitude of the forces as far as theory is concerned. This method of design will always give better results than the present method for one and the same system of forces; but just because this new method is valuable, the true set-up of force should be determined more accurately, and to accomplish this the continuation and extension of the author's tests, or perhaps the introduction of new tests, is necessary.

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DISCUSSIONS

FLEXIBLE "FIRST-STORY" CONSTRUCTION FOR EARTHQUAKE RESISTANCE

Discussion

BY H. A. WILLIAMS, ASSOC. M. AM. SOC. C. E.

H. A. WILLIAMS,[12] ASSOC. M. AM. SOC. C. E. (by letter).[12a]—The assumption is introduced in this paper that, if the first story of a building is made flexible, the remainder of the superstructure can be considered a rigid mass in so far as the dynamic analysis is concerned. The writer has made a number of shaking-table experiments in the Vibration Laboratory at Stanford University with two modifications of a dynamic model of a 17-story building (originally designed and built by Professor L. S. Jacobsen) to ascertain to what extent the author's assumption is valid.

For brevity in the discussion, the nomenclature will be given at once:

$e =$ elastic or spring constant (the same as that used by the author); e_1 is the constant for the first story; e_2, that for the second story; etc.

$\gamma = \frac{e_2}{e_1} =$ ratio of flexibility.

$T_n =$ fundamental period of vibration of model, in seconds, when the superstructure is not considered rigid. It is approximately equal to:

$$T_n = 2\,\pi\,\sqrt{\frac{\Sigma\, W\, \Delta^2}{g\, \Sigma\, W\, \Delta}} \qquad \text{(35)}$$

in which, W is the weight of one floor of the model, in pounds, and Δ is the horizontal deflection of that floor from the position of static equilibrium when the entire model is subjected to a horizontal force equal to the pull of gravity.[13]

$T_r =$ fundamental period of model, in seconds, when superstructure is considered rigid. It is equal to:

$$T_r = 2\,\pi\,\sqrt{\frac{m}{e_1}} \qquad \text{(36)}$$

in which, m is the mass of the entire superstructure.

$T_t =$ period of the shaking-table, in seconds.

NOTE.—The paper by Norman B. Green, Esq., was published in February, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: In May, 1934, by Messrs. Lee H. Johnson, Edward J. Bednarski, and Merit P. White and Paul L. Kartzke; and August, 1934, by Howard G. Smits, Esq.

[12] Instructor in Civ. Eng., Stanford Univ., Stanford University, Calif.

[12a] Received by the Secretary October 15, 1934.

[13] "Vibration Problems in Engineering", by S. Timoshenko, p. 63.

The first model was dynamically similar, as far as shear distortions are concerned, to a symmetrical 17-story steel frame office building of conventional design. The 8-story model was dynamically similar to the same building with the upper nine stories removed. Frictional effects in the models were not dynamically similar to those in the actual buildings. The weight per floor for Floors 2 to 13, inclusive, was 2 650 500 lb. The next three floors averaged approximately 2 500 000 lb, and the roof, 2 120 000 lb. The elastic constant, e_2, was 57 000 000 lb per in.; the constant, e_{16}, was 27 000 000 lb per in. The intermediate constants varied approximately as a straight line between these two values. The approximate ratio of height to width was 3 to 2 for the 17-story building and 2 to 3 for the 8-story building.

The shaking-table on which the model was placed, weighed more than $3\frac{1}{2}$ tons. Hence, the motion of the model did not affect the motion of the table, which was anchored at one end by a set of springs, in such a manner that it would be set in motion by dropping a pendulum against a bumper spring on the other end. After the impact, the table had a damped harmonic motion, as shown by the lower curve in Fig. 13. The period of of motion was changed for some of the tests by changing the anchor springs. The amplitude was varied by changing the height of the pendulum drop.

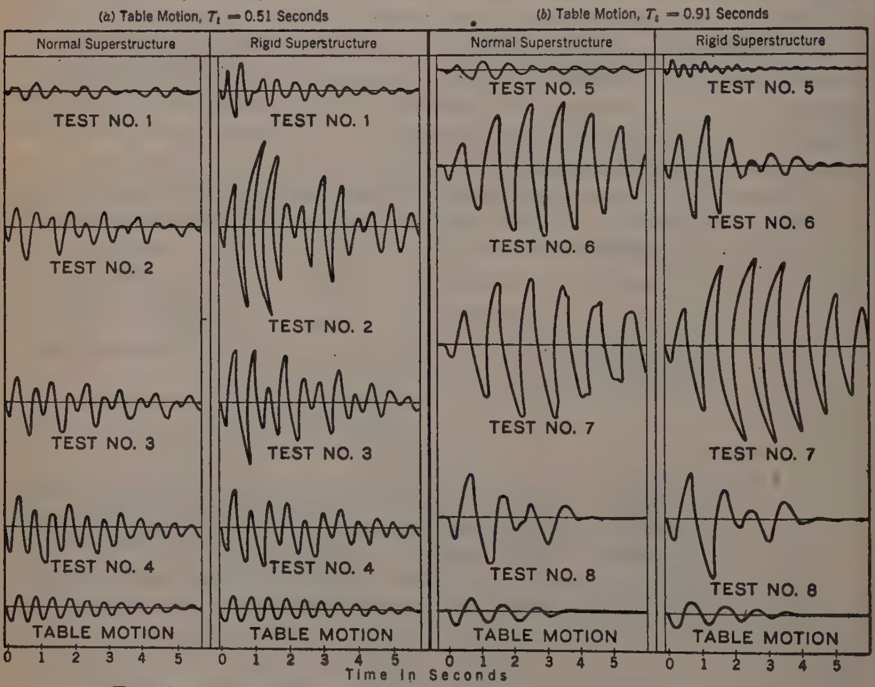

FIG. 13.—TESTS RESULTS FOR A MODEL OF A 17-STORY BUILDING.

The apparatus was arranged so that one record giving the relative motion of the second floor of the model with respect to the table and another giving the motion of the table with respect to the laboratory floor, were obtained simultaneously. In terms of an actual building, these records correspond to the relative motion of the second floor with respect to the ground and the absolute motion of the ground, respectively. In the test records, the table amplitude should be divided by 2 when compared quantitatively to the model amplitude.

For Test No. 1, the value of e_1 for the model corresponded to an elastic constant of 60 800 000 lb per in. in the actual building. Hence, the flexibility ratio was $\gamma = \frac{e_2}{e_1} = 0.95$ (see Table 3(*a*)). This ratio of flexibility would correspond to that found in some conventional buildings of this height.

TABLE 3.—COMPUTED PERIODS OF VIBRATION OF BUILDING MODELS

17-STORY BUILDING (SEE FIG. 13)						8-STORY BUILDING				
Test No.	Ratio of flexibility, γ	Periods of Vibration				Test No.	Periods of Vibration			
		Normal Structure		Rigid Structure			Normal Structure		Rigid Structure	
		T_n, in seconds	Ratio, $\frac{T_n}{T_t}$	T_r, in seconds	Ratio, $\frac{T_r}{T_t}$		T_n, in seconds	Ratio, $\frac{T_n}{T_t}$	T_r, in seconds	Ratio, $\frac{T_r}{T_t}$
(*a*) PERIOD OF TABLE MOTION, T_t, EQUALS 0.51 SECONDS										
1......	0.95	0.75	1.47	0.30	0.59	9.....	0.32	0.63	0.18	0.35
2......	6.10	0.94	1.84	0.67	1.31	10.....	0.56	1.10	0.43	0.84
3......	9.60	1.07	2.10	0.85	1.67	11.....	0.60	1.18	0.55	1.08
4......	38.80	1.82	3.57	1.65	3.23	12.....	1.10	2.16	1.00	1.96
(*b*) PERIOD OF TABLE MOTION, T_t, EQUALS 0.91 SECONDS										
5......	0.95	0.75	0.82	0.30	0.33	13.....	0.32	0.35	0.18	0.20
6......	6.10	0.94	1.03	0.67	0.74	14.....	0.56	0.62	0.43	0.47
7......	9.60	1.07	1.18	0.85	0.93	15.....	0.60	0.66	0.55	0.60
8......	38.80	1.82	2.00	1.65	1.81	16.....	1.10	1.21	1.00	1.10

The computed fundamental period of the model for this value of γ was $T_n = 0.75$ sec. The computed period of the second mode of vibration was approximately 0.25 sec. The table was then given the motion indicated by the record at the bottom of Fig. 13, and the corresponding motion of the second floor of the model was recorded with the superstructure in the normal condition of flexibility. The superstructure of the model was then braced so that it acted as a rigid mass, and the same test was repeated. The motion of the second floor for this condition is shown by the right-hand record for Test No. 1.

The first story was then made more flexible and the same procedure was followed for Test No. 2, etc. Since the table motion was the same for all tests in Fig. 13(*a*), it is shown only once, at the bottom.

Repetition of the same tests for the 17-story model, using a different table motion, gave the results shown in Fig. 13(*b*). The upper nine stories were then removed and the same procedure was followed for the 8-story model. It should be noted that an 8-story building with such large elastic constants would be quite stiff for the height involved.

The test records in Fig. 13 indicate that the second-floor motions with the two types of superstructure are not alike for this particular 17-story building when γ is less than approximately 40; and, even then, they are not exactly alike. The second mode of vibration whan γ was 38.8 was approximately 0.36 sec. It is quite probable that, if the table motion happened to have the same period as the second mode of vibration of the model, the favorable comparisons obtained in Tests Nos. 4 and 8, Fig. 13, would not hold.

The test records for the 8-story model are not shown, but they also give little evidence in favor of assuming a rigid superstructure in the 8-story building when γ is much less than 40.

Tests Nos. 1 to 4 might lead one to conclude that the assumption of a rigid superstructure is on the side of safety, because all the curves on the right-hand side of Fig. 13(*a*) have larger amplitudes than those on the left-hand side. Tests Nos. 6, 10, and 15 indicated that such a conclusion is erroneous. In general, if the ratio, $\frac{T_n}{T_t}$, is approximately 1.00 (exact resonance for the fundamental mode of vibration) and, at the same time, is closer to unity than $\frac{T_r}{T_t}$, the amplitude of the second-floor motion for the normal superstructure will be larger than when the superstructure is assumed to be rigid.

The experiments have covered a comparatively narrow range. Definite conclusions applying to all types of structures cannot be drawn from these few tests, or from any theoretical step-by-step method of analysis. In general, it would appear that if the elastic constants and floor weights are of the order of those given herein, there is little reason to believe that the author's method of analysis should apply to buildings higher than eight or ten stories unless the flexibility ratio, γ, is at least 40. This ratio can probably be somewhat less for buildings of fewer stories if the superstructure is fairly rigid. It probably should be increased if the building exceeds fifteen stories.

The writer understands that Mr. Green has in mind flexibility ratios as large as 50 or 60. The period of 3.58 sec for the building which he used in his illustration would indicate as much. The practicability of using a flexibility ratio of even 40, of course, is open to debate. Observation of the model on the shaking-table makes one dubious of the success of such a building. A small movement of the ground can set up a rather large relative motion of the structure even though the system is considerably removed from the resonance condition. Hence, certain practical difficulties must be overcome to prevent damage to the flexible story from every slight earth shock.

Some engineers believe it is practical to construct the first-story walls and ceiling so that the motion of the superstructure will result in no damage to the flexible story. If such a building has a flexibility ratio of 40, or more, its behavior in wind storms would have to be determined. It has also been suggested that hollow tile walls, for instance, could be installed in the flexible story to prevent damage from small ground motions and also to stiffen the building against gusts of wind. In a severe quake, these walls would soon shatter and allow the flexible story to come into full play. Until the tile walls are badly shattered, it is quite possible that vibrations will be set up in the superstructure and, also, that the elastic constant for the flexible story will be non-linear. These factors should be borne in mind when interpreting any mathematical analysis of such a building.

If it is granted that large flexibility ratios are practicable, Mr. Green's method of analysis, when used by one thoroughly familiar with the dynamic principles involved, should give some indication of the action of the building in an earthquake. Obviously, any quantitative results of such an analysis should be accepted with strong reservations. Even though the designer is skilful in simulating probable earth motions, there are several important factors which he would usually neglect because of lack of time or lack of reliable information.

For example, if a building is tall and narrow, direct stress in the columns and load-bearing walls will result in a certain amount of flexibility in the superstructure. This factor can be included when computing the elastic constants. If it is neglected, the flexibility ratio will be too high. Moreover, the rotation of the ground at the foundation line affects the motion of any building, the effect being dependent on the width and height of the building and on the nature of the ground below the foundation line. Another factor is damping friction which decreases the amplitude of motion. Little is known at present about the magnitude of these two factors as far as buildings are concerned.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

A GENERALIZED DEFLECTION THEORY FOR SUSPENSION BRIDGES

Discussion

By MESSRS. FREDRICK, VOGT, LEON S. MOISSEIFF, AND
A. MITCHELL AND G. T. PARKIN.

FREDRIK VOGT,[41] ASSOC. M. AM. SOC. C. E. (by letter).[41a]—A reference to the studies[42] of the late Hans H. Rode, M. Am. Soc. C. E., is pertinent to this paper.

Following Equation (9), the author states that "in the Deflection Theory, influence lines (in the ordinary sense) cannot be used". Professor Rode proved that such lines can be used for computation of the additional cable stress due to live load and the other effects of live loads, such as bending moments, etc. No doubt, the use of influence lines is limited to the case of proportionality between load and effect, and no exact proportionality exists for suspension bridges. However, on long suspension bridges, the dead load is large as compared with the live loads, and within the range from zero to full live load the proportionality is rather close, whereby the use of influence lines is made possible. Strictly speaking, one influence line should be computed for each particular value of the horizontal cable stress, H, between maximum and minimum, but Professor Rode found that sufficient accuracy was obtained by the use of two influence lines, one for a maximum, and one for a minimum, value of H (Tension H depending on temperature and load), between which a linear interpolation can be applied.

The importance of using influence lines, of course, is that the load lengths for maximum positive and negative moments, etc., can be determined directly instead of by trial as used by the author. Furthermore, it should be mentioned, that the method developed by Professor Rode can be applied for

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NOTE.—The paper by D. B. Steinman, M. Am. Soc. C. E., was published in March, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: May, 1934, by E. Pavlo, Esq., August, 1934, by Messrs. Jonathan Jones, A. Müllenhoff, H. Cecil Booth, Jacob Feld, and Glenn B. Woodruff, Howard C. Wood, and Ralph A. Tudor; and September, 1934, by Messrs. L. J. Mensch, A. A. Eremin, Hans H. Bleich, F. H. Frankland, Gustav Lindenthal, Julian W. Shields, A. W. Fischer, and J. M. Frankland.

[41] Prof. of Mechanics, Norwegian Univ. of Technology, Trondhjem, Norway.

[41a] Received by the Secretary August 15, 1934.

[42] "New Deflection Theory", by the late Hans H. Rode, M. Am. Soc. C. E., *Transactions*, Det Kgl. Norske Videnskabers Selskab (1930), No. 3. (In English.)

systems of any shape, statically indeterminate in any degree, and the stiffness may vary arbitrarily from panel to panel. Professor Rode takes into account the effect of introducing the deflection theory for the elastic theory by means of a "replacement system", which is identical with the original, except for the addition of certain elastic restraints, some or all of which may be statically indeterminate. This replacement system can be analyzed by the ordinary methods; that is, by the elastic theory. Since the replacement system is statically indeterminate to a high degree and the analysis, consequently, is rather complicated, Professor Rode also developed[41] methods for numerical computations.

LEON S. MOISSEIFF,[43] M. AM. SOC. C. E. (by letter).[43a]—The Deflection Theory has been known to American bridge engineers for some time and has formed the basis of the design of almost all important suspension bridges in this country for the last twenty-five years. It will be of interest to relate herein some of its early history in the United States.

In the summer of 1901, due to a combination of causes, a number of suspension rods in the main span of the Brooklyn Bridge failed. At that time this was the only crossing over the East River and carried most of the passenger traffic from Brooklyn, N. Y., to New York City. The Williamsburg Bridge was then in the process of construction. The uninterrupted travel over the Brooklyn Bridge was then of vital importance to the inhabitants of New York. Great public interest was aroused by the incident and an engineering investigation and report was made at the direction of the City's Corporation Counsel by the late Edwin Duryea, Jr., and Joseph Mayer, Members, Am. Soc. C E.[44] During the controversy on the behavior and safety of the Brooklyn Bridge which followed the publication of the engineering report, the writer worked on an analysis of the stresses and deformations of that bridge. He found that while there were no official measurements of the deflection of the bridge under load, such deflections as could be observed daily were of much less magnitude than would result from computations based on the Elastic Theory. Further consideration showed that, neglecting altogether the effect of the stiffening trusses on the deflection of the bridge and going back to the simple computations of the equilibrium polygons under dead load and under dead and live load, much closer agreement with the actual behavior of the bridge would result. It became apparent that the tacit neglect by the elastic theory of the effect of the dead load on the deformation of the bridge under live load was a serious error for large bridges. This is explained by the fact that the Rankine-Ritter theory was based on a rigid stiffening truss and thus logically introduced the error. It was later taken over without further scrutiny by the "fathers" of the elastic theory. Due to this, the latter theory is grossly in error for suspension bridges.

While working on the development of a more correct analysis, the writer directed his attention to the short treatment given by Melan to what he called

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[43] Cons. Engr., New York, N. Y.

[43a] Received by the Secretary September 24, 1934.

[44] *Engineering News*, October 10, 1901.

"The More Exact Theory."[45] Strangely, Melan did not put much emphasis on the practical results of the theory. The writer found that Melan's analysis of a one-span suspension bridge, taking into account the effect of the dead load on the deformation of the bridge under live load, gave solutions which closely approached actual conditions.

The Manhattan Bridge, as built, has a center span and two symmetrical side spans. To make a practical application of the theory to the design, it became the task of the writer to develop it for a three-span bridge and to establish the equations for the various loadings and temperature variations causing greatest stress. Bearing in mind the importance of the bridge which was designed for a congested live load of 16 000 lb per lin ft, the analysis was made with much hesitation and caution, and various methods were devised to furnish verifications of the computed results. Realizing that there can be no theory which, applied to a practical structure, will be absolutely exact, the writer subsequently denoted the developed theory by the name of "The Deflection Theory," having in mind that it is based on the deflections of the cable and of the truss. Thus, the Manhattan Bridge was designed in 1904 by the deflection theory, and may be considered the first modern suspension bridge.

Previous to that date, the writer in a discussion of the paper by the late Joseph Mayer, M. Am. Soc. C. E., entitled "The Stiffening System of Long-Span Suspension Bridges for Railway Trains," presented an application of the deflection theory to determine the temperature stresses in a three-hinged stiffening truss.[46]

In the beginning of 1909, when the Manhattan Bridge was approaching completion, the Commissioner of Bridges appointed Ralph Modjeski, M. Am. Soc. C. E., to report on the design and construction of the bridge. Mr. Modjeski engaged F. E. Turneaure, Hon. M. Am. Soc. C. E., for the analytical work. The writer submitted to the latter the fully developed theory and formulas used in the design of the main structure.[47]

In the "Final Report of the Board of Engineers on the Bridge over the Delaware River" in 1927 (see Appendix D), the writer presented the deflection theory as applied to that bridge.

As developed and published the deflection theory was adapted for single-span, and three-span, suspension bridges with stiffening trusses discontinuous over their supports, and all long-span suspension bridges built during the last twenty-five years have been so constructed. The engineers who designed the several bridges with continuous trusses, mentioned by the author, evidently did not think it worth while to utilize the deflection theory.

Attention is called to the fact that Professor Turneaure, whose work is cited in the paper, states:[48] "For this case [suspended side spans] the constants of integration are determined by equating the moments at the towers for main span with those for side spans, and the deflections at the towers equal to zero." This is the same method used in the paper.

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[45] "Eiserne Bogenbrücken und Hängebrücken, von J. Melan, 1888, Wilh. Engelmann, Leipzig, Germany.

[46] *Transactions*, Am. Soc. C. E., Vol. XLVIII (1902), pp. 422 *et seq.*

[47] In Mr. Modjeski's report, the theory is given in Appendix B. In the Ninth Edition, 1911, of "Modern Framed Structures," by J. B. Johnson, C. B. Bryan, and F. E. Turneaure, it can be found in the Chapter on "Suspension Bridges" under "Exact Methods of Calculation."

[48] "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, Ninth Edition, Pt. 2, John Wiley & Sons, 1911.

The author shows that some saving in materials and cost, or increased stiffness, could be realized by making the trusses continuous over the tower supports. He, therefore, has extended the formulas of the deflection theory to include continuous stiffening trusses and multiple tied spans. The work principle which is utilized in the derivation of the equations of the deflection theory lends itself well to expansion over these types. The author has succeeded in preparing a presentation of the theory which elegantly embraces the common types of suspension bridge as well as the one with continuous trusses and multiple tied spans.

In generalizing a theory which in practice has been found workable and sufficiently accurate for the required purposes, it is important to scrutinize the original assumptions on which the theory has been based. Usually, some of the assumptions are apparent and have been explicitly stated and some are implied and must be brought out by a trenchant analysis. It is in the character of science to guard its steps. The verity of deduced applications which are extended beyond common practice must be checked by quantitative studies and by physical observations. The application of the deflection theory to continuous stiffening trusses may serve to illustrate the limitations of general theory.

The deflection theory assumes that the effect of the elongation of the suspenders on the total work of the structure is negligible and that its effect on the value of the cable pull is insignificant. This assumption has been justified by computations in several instances as well as by observations on models. The effect of suspender elongation cannot, however, be neglected locally. Near the towers, where the suspenders are longest, the effect of their lengthening under live load on the stiffening truss cannot always be neglected and in long-span bridges it may seriously affect the stresses in the panels nearer the towers. Careful and extensive studies of the effect of suspender elongation on long spans have shown that the moments and shears in the trusses will increase considerably toward the tower ends and, in some panels, will require an increase of as much as 25% in sectional area of the chord members and diagonals. In shorter spans, this effect, of course, will be less, but it will not decrease proportionally. This has a pertinent bearing on the design of continuous trusses.

In the case of trusses hinged at the towers, the chord sections in the panels nearer the ends commonly over-run and the excess of section is in most instances sufficient to take care of the hanger effect. The writer has followed the practice of designing the rockers and diagonals near the ends for an excess stress of one-third to one-half. In the case of continuous trusses the heaviest chord stresses are at the towers, and an excess of section, due to over-run, will scarcely be available. The effect of suspender elongation, because of the stiffness of the truss due to continuity and heavy chord sections, will here be much more conspicuous and will require still larger sections.

Much of the theoretical saving may be nullified by it. It will be found desirable to study the effect of suspender elongation and tower shortening before reaching conclusions concerning continuous stiffening trusses.

In considering the behavior of continuous trusses, it should not be forgotten that the trusses are suspended from the cables for their dead load and that they must follow the deflections of the cable. The cable curve does not reverse its direction near the towers; the continuous truss, on the contrary, tends to do so. Such behavior must result in straining the truss unduly.

Another instance in which the assumptions of the deflection theory for the case of hinged trusses are sufficiently close is the use of a constant equivalent moment of inertia for each individual span. For discontinuous trusses, simple methods, based on various loadings, are used to complete the equivalent moment of inertia. These methods give close results. For continuous trusses the accurate determination of an equivalent moment of inertia is much more difficult and involved.

The writer does not wish to imply that practical problems will not be encountered in which continuity of the stiffening truss will be desirable. Small and light spans which commonly have an over-run of materials because of practical considerations may be given increased stiffness by continuity, and special conditions may make it advisable to resort to continuity in other instances.

To avoid misunderstandings, the writer calls attention to the fact that in the original publication of the deflection theory the values of the constants, C, represent lines or distances, while in the paper they represent moments, distance times H. The writer finds that the original notation is more helpful for the physical interpretation of the various terms.

For purposes of comparison the moment curves as computed by the elastic and the deflection theories shown in Fig. 3 are instructive. They prove again how much in error the elastic theory is for suspension bridges. It is timely to point out that the elastic theory may well be abandoned for suspension bridges and that attempts to tie it to the deflection theory, or any other advanced analysis, are of an atavistic nature.

In conclusion, the writer wishes to express his appreciation of the fine work done by Mr. Steinman in extending the deflection theory for continuous stiffening trusses and multiple tied spans.

A. MITCHELL[49] AND G. T. PARKIN,[50] JUNIORS, AM. SOC. C. E. (by letter).[50a]—A major contribution to the mathematical treatment of suspension bridges is contained in this paper. New developments in construction, together with the practical form of the author's method of designing, should bring about an increasing use of short-span highway suspension bridges of the continuous type.

Since "stresses producible by combinations of loadings cannot be found by adding algebraicly the respective stresses producible by the component loadings" (see following Equation (9)), it is desirable to have general formulas for the load functions, A, B, and G. Equations of the suggested form are readily reducible to the special cases treated by the author by noting that $e^{-cl}\,(1 + d) = 1 - d$ and $e^{cl}\,(1 - d) = 1 + d$. Thus, for G:

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[49] Senior Draftsman, State Highway and Public Works Comm., Raleigh, N. C.

[50] Designer, State Highway Comm., Bridge Dept., Raleigh, N. C.

[50a] Received by the Secretary September 24, 1934.

$$G_{1z} = \frac{1 - d}{4\,d}\Big\{P_k\,(e^{-ck} + e^{ck} - 2) - P_m\,(e^{-ck} + e^{ck} - e^{-c(l-z)} - e^{c(l-z)}) - P_z\,(e^{-c(l-z)} + e^{c(l-z)} - 2\,e^{cl})\Big\} \qquad (104)$$

$$G_{2z} = \frac{1 + d}{4\,d}\Big\{- P_k\,(e^{-ck} + e^{ck} - 2) + P_m\,(e^{-ck} + e^{ck} - e^{-c(l-z)} - e^{c(l-z)}) + P_z\,(e^{-c(l-z)} + e^{c(l-z)} - 2\,e^{cl})\Big\} \qquad (105)$$

$$G_{3z} = \frac{1}{2}\Big\{- P_k\,(e^{-ck} + e^{ck} - 2) + P_m\,(e^{-ck} + e^{ck} - e^{-c(l-z)} - e^{c(l-z)}) + P_z\,(e^{-c(l-z)} + e^{c(l-z)})\Big\} \qquad (106)$$

$$G_{1m} = \frac{1 + d}{4\,d}\Big\{P_k\,(e^{-c(l+k)} + e^{-c(l-k)} - 2\,e^{-cl}) - P_m\,(e^{-c(l+k)} + e^{-c(l-k)} - e^{-cz} - e^{cz}) - P_z\,(e^{-cz} + e^{cz} - 2)\Big\} \qquad (107)$$

$$G_{2m} = \frac{1 - d}{4\,d}\Big\{- P_k\,(e^{c(l-k)} + e^{c(l+k)} - 2\,e^{cl}) + P_m\,(e^{c(l-k)} + e^{c(l+k)} - e^{cz} - e^{-cz}) + P_z\,(e^{-cz} + e^{cz} - 2)\Big\} \qquad (108)$$

$$G_{3m} = \frac{1}{2}\Big\{- P_k\,(e^{-ck} + e^{ck} - 2) + P_m\,(e^{-ck} + e^{ck})\Big\} \qquad (109)$$

$$G_{1k} = \frac{1 - d}{4\,d}\Big\{- P_k\,(e^{c(l-k)} + e^{-c(l-k)} - 2\,e^{cl}) + P_m\,(e^{c(l-k)} + e^{-c(l-k)} - e^{cz} - e^{-cz}) + P_z\,(e^{cz} + e^{-cz} - 2)\Big\} \qquad (110)$$

$$G_{2k} = \frac{1 + d}{4\,d}\Big\{P_k\,(e^{c(l-k)} + e^{-c(l-k)} - 2\,e^{-cl}) - P_m\,(e^{c(l-k)} + e^{-c(l-k)} - e^{cz} - e^{-cz}) - P_z\,(e^{cz} + e^{-cz} - 2)\Big\} \qquad (111)$$

and,

$$G_{3k} = P_k \qquad (112)$$

in which, in addition to the notation of the paper, the subscripts, z, m, or k, are added to P and G to indicate the portion of the span to which they apply (see Fig. 1).

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DISCUSSIONS

SAND MIXTURES AND SAND MOVEMENT IN FLUVIAL MODELS

Discussion

By Messrs. V. V. Tchikoff, Morrough P. O'Brien and Bruce D. Rindlaub and Herbert D. Vogel.

V. V. Tchikoff,[35] M. Am. Soc. C. E. (by letter)[35a].—A good example of laboratory investigation is offered in this excellent paper. There are serious doubts, however, as to the theory of tractive forces, or "drag" theory, itself, as well as the quantitative transferability of experimental results. The "drag" force is simply a component, parallel to the slope, of the weight of a vertical column of water of height, d, in contact with a unit area of bed of a channel with uniform flow as expressed by Equation (3). This formula is considered by the "drag" theory as a constant relation between the slope and the depth for the same bed-load material. This theory is not generally recognized. "The imperfection in this concept is evident, and it is clear that the phenomena as conceived by du Boys was a scheme somewhat lacking in the real course of things", states Professor Francesco Marzolo.[36] Professor Reynold considers the force exerted by the current on sand as the function of the velocities. Professor A. H. Gibson states that the erosive power of water varies as the square of its velocity.[37] This "drag" theory has not even been mentioned in many important books. The theory has the following limitations:

1.—It presupposes infinite width of channel and, according to Schoklitsch,[38] an additional coefficient is required for channels in which the ratio of width to depth is less than thirty. The "drag" theory does not take into consideration the type and nature of the cross-section of the channel.

Note.—The paper by Hans Kramer, Assoc. M. Am. Soc. C. E., was published in April, 1934, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1934, by Messrs. John Leighly, Paul W. Thompson, and Gerard H. Matthes; and September, 1934, by Messr. R. H. Keays, and F. T. Mavis.

[35] Cons. Engr., New York, N. Y.

[35a] Received by the Secretary August 11, 1934.

[36] "Hydraulic Laboratory Practice", p. 755.

[37] "Hydraulics and Its Application', Fourth Edition, p. 340.

[38] "Hydraulisches Rechnen", von Weyrauch und Strobel (1930), p. 79.

2.—Schaffernak[39] considers the du Boys law only applicable for certain sizes of grain, namely, for homogeneous materials with a grain size up to about 10 mm.

3.—According to the author the slopes must be "usual moderate slopes", as he stated in establishing his law of constant critical tractive forces.

4.—The type of the flow does not affect the "drag" theory, and it is considered applicable for laminar, turbulent, shooting, or streaming flows.

These four limitations cover practically every phase of hydraulic phenomena of flow and are so serious, in fact, that they annul the entire theory. There are direct contradictions between the requirements for hydraulic similitude of channels and those fulfilling the "drag" theory. As the writer had occasion to state in another connection,[40] the first requirement for similitude of turbulent flow is the same kinetic factor. This is nothing but the fulfillment of Froude's law. The kinetic factor, or twice the ratio of kinetic energy to the hydraulic radius, gives only the physical and quantitative state of turbidity. Expressing the velocity through the kinetic factor:

$$V = \sqrt{g K_r R} \quad \text{.......................(15)}$$

Comparing with Chezy's equation:

$$S C^2 = g K_r \quad \text{.........................(16)}$$

in which, C = Chezy's coefficient and $K_r = \dfrac{V^2}{g\,R}$ = the kinetic factor. The right-hand part of Equation (16) is constant for the same hydraulically similar channels.

Equation (16) gives an important relation between the slope and elements of the cross-section. The same relation is expressed in Equation (3):

$$S d = \text{a constant} \quad \text{.......................(17)}$$

To meet the requirements for hydraulic similitude and the "drag" theory, both Equations (16) and (17) must hold, or:

$$C = \text{constant} \times d \quad \text{.......................(18)}$$

This is the new and non-existent expression for Chezy's coefficient, C. The search for the relation between the slope and other elements in some stable alluvial channels which are hydraulically similar, led Dr. H. E. Hurst to the following equation for the Lower Chenab Irrigation Project[41]:

$$S = 116 \times 10^{-6} \left(1 + \frac{2.12}{R}\right) \quad \text{.................(19)}$$

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[39] "Hydraulic Laboratory Practice", p. 428.

[40] *Proceedings*, Am. Soc. C. E., April, 1934, p. 557, *et seq.*

[41] *Minutes of Proceedings*, Inst. C. E., Vol. 229, Session 1929–30, Pt. I, p. 328.

If Equation (19), with $K_r = 3.98\%$ (its value for the Lower Chenab Irrigation Project,[42] see Table 5) is substituted in Equation (16):

$$C = \frac{105.4}{\sqrt{1 + \frac{2.12}{R}}} \qquad (20)$$

TABLE 5.—GENERAL HYDRAULIC FACTORS; BRITISH INDIAN IRRIGATION PROJECTS

Name of irrigation project (1)	Numbers of observations (2)	Mean kinetic factor, K_r (percentages) (3)	Coefficient of hydraulic similitude, Y (4)	Hydraulic number, H (5)	Difference from mean (percentages) (6)	TRACTIVE FORCE, IN GRAMS PER SQUARE METER: Maximum (7)	Minimum (8)
Upper Bari Doab......	22	4.15	7.20	35.3	—12.8	381	58
Lower Chenab........	35	3.98	8.86	44.4	+ 9.6	537	112
Godaveri West Delta................	15	2.04	5.55	38.8	— 4.2	264	88
Mean weighted....	....	....	....	40.5	0.0	....	....

This new type of equation for the Chezy coefficient is most correct compared with all existing formulas, such as, those of Kutter, Bazin, Manning, etc. All of the latter are entirely different from Equation (18) expressing the "drag" theory. In other words, the channel fulfilling the "drag" law will not be hydraulically similar. (A formula of the same type as Equation (20) has been suggested by the late W. M. Penniman,[42] M. Am. Soc. C. E. Equation (20) has numerical coefficients corresponding to the coefficient of roughness, $n = 0.0225$.)

The idea of the kinetic factor facilitates the judgment of the state of turbulency of flow. Table 6 gives the kinetic factor for the lower limit of tractive force and the adjacent readings (Column (4)) for Series II of the author's data. The kinetic factor varies from 34% for a slope of 1:1 000 to 80% for a slope of 1:400, being more constant for smaller slopes.

TABLE 6.—GENERAL HYDRAULIC FACTORS; SAND, SERIES II, FROM AUTHOR'S DATA

Slope	Reading number	Tractive force, in grams per square meter	Kinetic factor (percentages)	Coefficient of hydraulic similitude	Hydraulic number	Ratio of width to depth
1:400....................	4	38.1	80	1 263	1 413	....
	4a	41.0	81	....	....	49
	5	49.5	77	896	1 022	....
1:600....................	5	40.5	43	501	759	....
	5a	41.7	42	....	....	32
	6	44.2	40	443	703	....
1:800....................	4	33.7	36	405	675	....
	4a	35.8	39	....	....	28
	5	37.9	40	362	575	....
1:1 000.................	4	36	34	264	455	....
	4a	37.2	34	....	....	22
	5	39.6	34	235	405	....

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[42] *Engineering News-Record*, May 10, 1934, p. 608.

Mr. Gerald Lacey[43] gives the approximate equation for the relation between his silting factor and the diameter of the predominant type of sand, which expressed through the kinetic factor, is:

$$D = 8.6\ K_r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

in which, D, is the diameter of the sand grains, in inches. Sand of Series II had the maximum diameter of 1.77 mm = 0.07 in. and a median diameter of 0.51 mm = 0.02 in. Substitution of these diameters into Equation (21) gives the corresponding value of the kinetic factors as 9.0 and 4.8%, respectively. Comparing this value of the kinetic factor with that of the flume in the paper, it is seen that the latter is several times greater. In other words, maintaining hydraulic similitude, the bed-load material in the prototype corresponding to the "geschiebe" in the flume, must be considerably coarser. Its size of grain, calculated according to the kinetic factor in the flume, will be, for $K_r = 0.80$, — $D = 5.5$ in., and for $K_r = 0.34$, — $D = 1$ in.

TABLE 7.—GENERAL HYDRAULIC FACTORS; ALLUVIAL CANALS AND MISSISSIPPI RIVER

Reference No.	Name	Description of channel	Discharge per second, in feet	Hydraulic radius, in feet	Mean velocity per second, in feet	Hydraulic grade; 5×10^{-4}	Kinetic factor (percentage)	Coefficient of hydraulic similitude	Hydraulic number	Tractive force, in grams per square meter
123a....	Bear River...	Silt..........	109.56	1.86	2.36	2.7	9.30	17.0	56	160
146a....	Logan and Hyde Park..	Sand.........	23.55	1.40	1.08	1.5	2.59	8.6	53	101
168a....	Louden......	Clean sand...	62.00	1.52	1.66	3.8	5.63	17.7	75	173
169a....	Mesa lateral..	Fine silt bed..	40.32	1.66	1.47	2.6	4.04	8.8	44	116
204.....	Fullerton.....	Loose sand bed	15.72	1.03	1.14	4.93	3.92	14.4	73	173
170a....	Rio Grande lateral.....	Gravel.......	146.60	1.42	3.86	22.0	32.6	51.2	90	980
210.....	Parley's Ditch lateral.....	Sand bed.....	4.44	0.66	1.20	11.7	6.78	15.5	60	264
201a....	Rocky Ford..	Loose sand bed	41.20	1.52	1.68	5.8	5.74	11.7	49	299
214a.....	Rio Grande lateral.....	Gravel.......	380	1.87	4.66	36.6	36.1	58.2	97	2 087
Mississippi River	....	....	1 125 000	51.00*	4.76	0.44	1.38	8.5	72	684
Vogel's model..	....	....	0.0828	0.255*	0.355	10.56	1.53	5.0	40	82

* Mean depth.

To give a concrete idea as to the value of the kinetic factor in canals. Table 7 is given, based on some alluvial canals studied by Fred C. Scobey, M. Am. Soc. C. E.[44] The last two lines of Table 7 give data on the Mississippi River and its model according to Herbert D. Vogel, Assoc. M. Am. Soc. C. E.[45]

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[43] "Stable Channels in Alluvium", by Gerald Lacey, *Minutes of Proceedings*, Inst. C. E., Vol. 229, Pt. I, p. 283.

[44] *Bulletin No. 194*, U. S. Dept. of Agriculture.

[45] "Geometric *Versus* Hydraulic Similitude", by Herbert D. Vogel, Assoc. M. Am. Soc. C. E., *Civil Engineering*, August, 1932, p. 471.

Comparison of Tables 6 and 7 in relation to the kinetic factors and tractive forces shows:

1.—Only two cases (Nos. 170*a* and 214*a* in Table 7), the kinetic factor being 32.6 and 36.1, more or less approach the same value in the flume. There is gravel in both cases. This proves again that the bed-load material in the prototype must be considerably coarser in order to correspond to the same material in the flume.

2. —The tractive forces in Table 7 are higher in general than in the flume, but all channels are not scouring. Three instances are especially interesting. Two are referred to the same channel with a gravel bed, in which, $T = 980$ and 2 087 g per sq m, and the third is the Mississippi River, with $T = 684$. All three values are much greater than the tractive forces in the paper.

The author suggests, as the most practical procedure, that the bottom silt taken from the prototype be tested in the flumes, but such "geschiebe" in a model flume will correspond to a much coarser material in the prototype. In designing the model of a channel with a different kinetic factor, it is necessary to take the corresponding bed-load material and, if Equation (21) is more or less correct, the ratio of the diameters of the "geschiebe" must be equal to the kinetic factor, squared.

Columns (7) and (8) of Table 5 show the maximum and minimum values of tractive forces. These three stable alluvial channels, as already stated, are hydraulically similar. Each of them transports its own specific sand corresponding to its kinetic factor. The tractive forces vary greatly, usually having a greater value in the larger channels. The maximum values are again considerably larger than in the flume, but scouring has no place.

The author referred to Gilbert's conclusion that with equal width-depth ratios, the result of experiments with a flume are, in general, quantitively transferable to channels of different trace. This means that if the cross-sections of the model and the prototype are a rectangle, they are geometrically similar; but the cross-sections of models of hydraulically similar channels should be distorted[40, 45] and this distortion must be given consideration because the preservation of hydraulic similitude depends not only on the same kinetic factor, but also on the form of the cross-section. To express this dependence the writer suggests the following equation[40]:

$$Q = Y\,R^3 \qquad \text{(22)}$$

in which, Y is a constant for hydraulically similar channels and may be termed the "coefficient of hydraulic similitude".

Another interesting relation between the kinetic factor, K_r, and the coefficient, Y, is:

$$H = \frac{Y}{K_r^{0.5}} \qquad \text{(23)}$$

Of course, Y and K_r being constant, the coefficient, H, is constant, too, for the same hydraulically similar channels. However, this last coefficient (termed the "hydraulic number") presents an interesting parameter.

Table 5 gives the value of H for three British Indian irrigation projects in which the channels are hydraulically similar. The hydraulic numbers for these three different projects with stable alluvial channels are about the same (approximately 40.0). This naturally suggests the idea that it is quite probable that all stable alluvial channels have the same hydraulic number. Stable alluvial channels are understood to be non-silting and non-scouring, ideal channels with a uniform flow of water and a full silt load. The value of the hydraulic numbers is computed for channels given in all the tables. It is of special interest to note that Lieut. Vogel's model has the same exact value of hydraulic number (40), while all others in Table 7 have larger values. The value, 40, appears to be a limit. All channels in Table 7 are non-scouring, and if the value of the hydraulic number in the alluvial stable channel is less than 40, it may be assumed that they have scouring tendencies when the kinetic factor corresponds with the silt material.

In data concerning the scouring of some drainage channels observed by Charles E. Ramser,[46] M. Am. Soc. C. E. (Table 8), the hydraulic number changes from 14.9 to 22.4 (less than 40) and the author's data differ completely from Tables 5, 7, and 8.

TABLE 8.—GENERAL HYDRAULIC FACTORS; ERODING DRAINAGE CHANNELS

Stream No.	Observation No.	Name	Description of channels	Discharge per second, in cubic feet	Hydraulic radius, in feet	Mean velocity, in feet per second	Slope, 5 × 10^{-4}	KINETIC FACTOR (PERCENTAGES)		COEFFICIENT OF HYDRAULIC SIMILITUDE		HYDRAULIC NUMBER		Tractive force, in grams per square meter
								Actual	Mean	Actual	Mean	Actual	Mean	
1	1*	Mud Creek, near Tupelo, Miss.	Alluvial, sandy, wax-like clay.	794	5.6	3.6	....	.....	.....	.....	.....	.....	.....	.....
	2			914	5.6	3.6	....	7.2	.....	5.2	.....	19.4	.....	.....
	3			1 221	6.5	4.0	....	7.6	.....	4.5	.....	16.1	.....	.....
	4			1 280	6.0	3.8	3.98	7.4	7.4	5.9	5.2	21.8	19.1	904
2	1*	Twenty-Mile Creek, near Baldwyn, Miss.	Wax-like clay, loam, containing considerable sand.	611	5.0	3.5	....	.....	.....	.....	.....	.....	.....	.....
	2			1 758	7.2	5.8	....	14.3	.....	4.7	.....	12.5	.....	.....
	3			2 402	7.4	4.7	11.6	7.4	10.8	4.7	4.7	17.3	14.9	2 996
3	1*	Chawappah Creek, near Shannon, Miss.	Sandy loam at top to wax-like clay at bottom.	582	4.9	3.5	....	.....	.....	.....	.....	.....	.....	.....
	2			1 354	6.6	4.35	....	7.6	.....	4.7	.....	17.2	.....	.....
	3			1 392	6.1	4.1	....	8.7	.....	6.1	.....	20.8	.....	.....
	4			1 456	5.9	4.0	9.29	8.4	8.3	7.1	6.0	24.4	20.8	1 910
11	1*	South Fork, Deer River, near Jackson, Tenn.	Firm, wax-like clay.	1 620	7.0	4.0	....	.....	.....	.....	.....	.....	.....	.....
	2			1 740	7.1	4.1	....	7.3	.....	4.9	.....	18.0	.....	.....
	3			2 342	8.2	4.5	5.15	7.8	7.5	4.2	4.5	15.2	16.6	1 388
13	1*	North Fork, Deer River, near Trenton, Tenn.	Alluvial silty loam at top, heavy silty clay at bottom.	883	5.3	3.9	....	.....	.....	.....	.....	.....	.....	.....
	2			1 512	6.6	4.6	....	9.8	.....	5.3	.....	16.8	.....	.....
	3			1 838	6.6	4.5	....	9.75	.....	6.4	.....	20.4	.....	.....
	4			2 144	7.0	4.8	5.02	10.1	9.9	6.3	6.0	19.6	18.9	1 176
15	1*	Sugar Creek, near Henderson, Tenn.	Light-yellow clay, very tenacious and much less easily eroded than soil of Stream No. 11.	345	3.7	3.27	....	.....	.....	.....	.....	.....	.....	.....
	2			400	3.8	3.3	....	8.7	.....	7.3	.....	24.8	.....	.....
	3			473	4.2	3.5	....	9.1	.....	6.4	.....	21.1	.....	.....
	4			513	4.3	3.55	8.38	9.1	9.0	6.4	6.7	21.3	22.4	1 232

* Original cross-sections at time of construction computed from dimensions of channels given in specifications. Mean values were computed without considering original cross-sections.

The phenomena of scouring does not occur in Nature as in a rectangular flume. First, it is known that the upper parts of the channel's banks, even under normal conditions, have a tendency to wash out, become round, and settle. The depths of water in the upper parts, and tractive forces, are less; but scouring occurs and settlement of washed material also occurs where the depth and tractive forces are greater.

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[46] "Erosion and Silting of Dredged Ditches", by Charles E. Ramser, M. Am. Soc. C. E., *Technical Bulletin No. 184*, U. S. Dept. of Agriculture, June, 1930.

Scouring and silting on a bank do not depend on the depth of the water, but on its velocity. Increased velocity will scour the banks of the channel and make it wider. A deep narrow stream has a tendency to widen itself, but the shallow wide stream will deepen some part of the channel to become narrower by silting the remainder. In both cases the stream inclines to adjust itself in a stable channel. Two critical conditions for erosion and silting, in the writer's opinion, depend upon the condition that the kinetic factor must correspond to the transporting material and hydraulic number. For any particular kinetic factor and silt there will be two values of the hydraulic number between which the channel will be stable. For channels having a non-cemented or non-packed bed of the same silt, carried by fully charged water, there will be the same hydraulic number (equal probably to about 40). In the case of a settled bed the difference may be considerable. The cohesion between the particles has to be overcome. The finer the settled materials, the larger the molecular bonds and skin friction will be; but the gravity action increases with the size of the particles. Therefore, clay and sand both need higher velocities to cause erosion than the coarse silt, which gives minimum erosion-velocity conditions.[47]

There is no turbulent flow with parallel filaments of water. The distribution of velocity over the cross-section is the all-important factor. Cross-currents greatly affect the phenomena of silting and scouring. Velocities at any point are constantly changing. Scouring and silting depend on the distribution of velocity along the cross-section (especially at the bottom) and not so much on their absolute value as on their relative changes. If a model is built correctly these relative changes of velocities should be about the same as in the prototype. This explains why hydraulically similar, alluvial, channels are capable of transporting the same material.

In studying any phenomena of flow in channels by the method of models, the first requirement is the hydraulic similitude of flow. Failing to fulfill this requirement and basing conclusions on the doubtful "drag" theory, the author's type of experiments seems to differ from the actual condition in Nature; they may have only a qualitative meaning. No doubt, however, the author's work (especially his new formula for tractive forces based on the silt graduation curve) is very important in laboratory practice.

MORROUGH P. O'BRIEN,[48] ASSOC. M. AM. SOC. C. E., AND BRUCE D. RINDLAUB,[49] ESQ. (by letter)[49a].—The rate of movement of bed-load in channels is known to increase rapidly with tractive force and, for this reason, a visual determination of the critical tractive force is probably sufficiently accurate for model work; but for establishing a general law, the more precise weighing method appears desirable. In this connection it should be mentioned that there is quantitative evidence indicating that each fraction of a mixture behaves approximately as it would if present alone, at least in the region of general movement. If this statement is true of a mixture acted upon by a tractive force too small to move the larger fractions, the critical tractive force for each fraction can be obtained approximately by analyzing the material caught in the weighing pan. For each sample, the tractive force acting is approximately the critical value for the largest particles caught.

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[47] "Problems in the Theory of River Engineering", by Herbert Chatley, Inst. C. E., Selected Engineering Papers No. 71, p. 17.

[48] Assoc. Prof. of Mech. Eng., Univ. of California, Berkeley, Calif.

[49] Berkeley, Calif.

[49a] Received by the Secretary August 25, 1934.

The use of depth times slope as the tractive force is perhaps sufficiently accurate for model work; but it is not a sound procedure since some resistance to flow is exerted by the side walls of the channel. The average tractive force on the side walls and bottom is:

$$T = w\,R\,s \quad\text{.........................(24)}$$

in which, s is the slope of the energy gradient (not the slope of either the bottom or the water surface); and R is the hydraulic radius. The tractive force on the bottom exceeds the average value in most cases and approaches, but does not equal, $w\,D\,s$.

In regard to the slope to be used in computing the tractive force, if sufficient data are not available to compute the slope of the energy gradient, the slope of the bottom is more nearly correct than that of the water surface, if the bottom is inclined downward in the direction of flow.

The author mentions a paper by Leighly[5] which describes a method of estimating the distribution of tractive force over the wetted perimeter of a channel. Another method that appears to be more sound can be developed from von Karman's theory of turbulent flow near a flat boundary surface. The essence of the theory underlying this method is that the velocity distribution, at a distance from the boundary greater than the thickness of the laminar layer, depends only upon the shearing or tractive force within the fluid. The equation for the tractive force is:

$$T = \rho \left[\frac{c\,(u - u_0)}{\log_e \dfrac{z}{z_0}} \right]^2, \quad\text{.....................(25)}$$

in which, ρ is the mass density; u and u_0 are the velocities at distances, z and z_0, from the boundary; and c is a coefficient found from experiment to have a value of 0.4. In order to apply Equation (25), velocities must have been measured at distances from the boundary which are small as compared with the dimensions of the cross-section. Application of this method involves the construction of velocity profiles along lines perpendicular to the bottom at each point.

An interesting possibility of this method is that it may be sufficiently precise to compute the value of the friction coefficient from the velocity distribution alone without measuring the slope of the energy gradient. The advantage of this method is that it would give the average value of n at one cross-section rather than the average over a finite reach.

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[5] "Toward a Theory of the Morphologic Significance of Turbulence in the Flow of Water in Streams", by John B. Leighly, Univ. of California Publications in Geography, Vol. 6, No. 1, pp. 1–22.

Assuming that T has been computed for a number of points around the wetted perimeter, the total resistance per unit length of channel is, $\int_0^p T\,dp$, in which, p is the wetted perimeter, and dp is an infinitesimal length along the perimeter. The total resistance is obtained by measuring the area under the curve of T as a function of p. The resistance as computed by the Manning equation is, $w\ A\ \frac{v^2\ n^2}{(1.49)^2\ R^{\frac{4}{3}}}$. Equating the two expressions for the resistance, and solving for n,

$$n^2 = \frac{(1.49)^2\ R^{\frac{4}{3}} \int_0^p T\,dp}{w\ A\ v^2} \qquad \text{(26)}$$

in which, A is the area of the cross-section, and v is the average velocity. Comparison of the value of n computed by this method with that obtained in the usual way will give a check on the validity of von Karman's theory as applied to open channel flow.

Returning to the critical tractive force and its relationship to the characteristics of a sand mixture, the writers are not convinced that the modulus, M, brings about an improvement in the correlation of the existing data. An analysis[50] of the data of Gilbert and others indicated that the critical tractive force can be related to the median diameter alone, with the same or less deviation of individual experimental points, and the relationship is much simpler to apply.

No quantitative theory of bed movement yet advanced adequately represents all the factors involved and, for the present at least, the characteristics of each sand to be used in a movable-bed model must be determined independently. A general theory is needed, but much additional experimental work will be required to develop one that can be applied with confidence. In the meantime, engineers designing models should use present equations with caution.

HERBERT D. VOGEL,[51] ASSOC. M. AM. SOC. C. E. (by letter)[51a].—The work of Captain Kramer in Germany, as reported in his paper, has served the past few years as no inconsiderable aid to all experimenters interested in the performance of model streams. Prior to the release of his data much conjecturing was needfully resorted to each time it became necessary to design a movable-bed river model; more recently, it has been possible to approach each problem from a standpoint of reason and logic.

Two statements of the author appear worthy at this time of special repetition and emphasis: (1) "* * * strict compliance with the laws of similitude must be waived in fluvial models in which bed movement is of predominating importance;" and (2) "since it [Equation (10)] was derived empirically and from limited data, its scope of application and probable accuracy are subject, of course, to obvious limitations."

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[50] "The Transportation of Bed-Load by Streams", by M. P. O'Brien, Assoc. M. Am. Soc. C. E., and B. D. Rindlaub, *Transactions*, Am. Geophysical Union, 1934.

[51] Lieut., Corps of Engrs., U. S. Army, Fort Leavenworth, Kans.

[51a] Received by the Secretary September 21, 1934.

Referring to the first of these considerations it is evident that the requirements for adequate bed movement in a fluvial model are at variance with those indicated for preservation of strict Froudian relationships. This is true because for any fixed model depth it is generally necessary to apply steeper slopes in order to produce a movement of the bed material similar to that of Nature, whereas to secure a velocity scale equal to the square root of the vertical scale, a model must be provided, in nearly every case, with flatter slopes than would result from normal distortion. Similarly, a large degree of distortion is conducive to greater bed movement, whereas little or no distortion is desirable from the standpoint of satisfying the Froudian law. Thus, the experimenter is faced with contradictory requirements, and the only solution is by way of deciding from an estimate of the situation the line of attack to pursue, keeping in mind the mission to be accomplished. This is generally an easy task.

The second point emphasized by the author, may cause some doubt in the minds of a few as to the reliability of his uniformity modulus. This is relatively unimportant, however. because future investigations will serve to reveal better values for it from time to time. The important fact is that a start has been made in the right direction and a line thus indicated for further research.

Investigations are being conducted currently in different parts of the world to determine specifically the effects on movement resulting from changing the specific gravity or the mixture of the bed material. Tests are also being made to determine volume rates of movement and the effects of turbulence, roughness, etc., on movement. All these have followed from the original investigation conducted and reported upon by Captain Kramer.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THE RESERVOIR AS A FLOOD-CONTROL STRUCTURE

Discussion

BY C. MAXWELL STANLEY, JUN. AM. SOC. C. E.

C. MAXWELL STANLEY,[7] JUN. AM. SOC. C. E. (by letter)[7a]. — Much information of a basic nature is assembled in this paper concerning the function and the design of reservoirs as flood-control structures. A number of methods of operating reservoirs and, correspondingly, of design, are presented, together with outlines of various manners of combined reservoir usage for flood control and other purposes. The methods of operation, and, hence, of design, of reservoirs for flood control are numerous. The ingenious methods which have been and will be developed to solve the mass of routine and tedious computations involved in the study of a reservoir system are equally numerous.

This paper has been written with a background largely of the Mississippi River, which, as the author points out, is undoubtedly the "No. 1" flood-control problem of this country. The size and scope of so large a flood-control project, as that required in the case of the Mississippi, create many situations and points of view which are not strictly applicable to projects of lesser size and scope. As an instance, consider the numerous references, in the paper, to the use of selected maximum floods of record in designing the controlling structures. While a considerable reliance on past records of stream flow may be justified when the flow comes from large areas and diversified sources, as in the case of the Mississippi, an equal reliance on past stream-flow records may cause grave error with drainage areas of only a few hundred or a few thousand square miles. In such instances, careful investigation of the rainfall probabilities must be made, as a single, heavy downpour may result in a run-off much greater than anything that has been experienced in the past. From the results of such an investigation, assisted, of course, by data on flood discharges of the past, a design flood may be determined

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NOTE.—The paper by George R. Clemens, Assoc. M. Am. Soc. C. E., was published in May, 1934, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[7] Cons. Engr. (Young & Stanley, Inc.), Muscatine, Iowa.

[7a] Received by the Secretary August 13, 1934.

which will form a basis for the proportioning of the controlling structures. Past records cannot be used blindly on any occurrence as far beyond the control of Man as rainfall and run-off.

What would have been the standards of design of controlling structures and the criterion of flood discharge on the Mississippi to-day, had the floods of 1927 been delayed until 1937 by some combination of hydrological circumstances beyond the control of Man?

Furthermore, small drainage areas do not offer the opportunities of predicting and forecasting run-off and river stage which are possible on a system the size of the Mississippi where reasonably accurate predictions of conditions may be made days and, sometimes, weeks in advance. In contrast, the floods on the small areas may develop within 24 hr and may come without warning as a result of a peculiar combination of rainfall and run-off conditions. The likelihood of sudden and sharp floods tends to rule against the intricate and completed schedules of reservoir operation which may be designed and used on a larger system where more time is available and where there is a competent staff to carry out the work of predicting the flood and scheduling the operation of the reservoirs. Indeed, the probability of the sudden and sharp flood peak on the small drainage area seems to demand, in most cases, a structure the operation of which is automatic and is not dependent upon the human element. This tends to eliminate the reservoir with an outlet controlled by gates and to favor the use of the retarding type of reservoir, or the complete elimination of reservoirs in favor of levees, channel improvements, etc.

Considerable controversy has raged between those advocating the use of reservoirs to limit the flood discharge and those advocating the use of structures, such as levees, floodways, etc., which increase the carrying capacity of the channel. Much of such controversy might be avoided by giving careful attention to the purpose of economical flood control.

The purpose of flood control, of course, is to prevent damage to property, and loss of life. This purpose is accomplished by constructing the necessary structures to assure that the maximum flood discharge at the point to be protected is within the carrying capacity of the channel through which the flood must pass. Obviously, this end may be obtained by decreasing the size of the flood peak, by increasing the channel capacity, or by a combination of these two methods.

The function of a reservoir is to decrease the flood peak passing a given point, or points, and the function of channel improvements, levees, floodways, etc., is to increase the carrying capacity of the channel. Theoretically, suitable protection might be obtained by the use of either reservoirs alone or by channel improvements alone, provided, of course, that suitable reservoir sites are available. The reservoirs must be sufficiently large to limit the flood flow to the capacity of the unimproved channel, or the channel must be of sufficient magnitude to provide a channel capacity capable of carrying the maximum unregulated flow. In many cases, one of these solutions may provide the most practical and most economical answer to the problem, but in many cases a combination of the two methods (that is, the use of both reservoirs and channel improvements) may provide a more economical solution.

The proportion in which they should be combined is largely an economic problem which must be solved for each project. As an example, assume the very simple hypothetical case shown in Fig. 12 of a flood-control project to protect a single city from a river of relatively small drainage area. The maximum flood discharge from this area is assumed to be 100 000 cu ft per sec. On the water-shed of this river, it is possible to develop a single reservoir that will regulate the flow from 70% of the drainage area. The existing channel through the city is capable of carrying 40 000 cu ft per sec, or 40% of the designed flood peak for the project without further improvements.

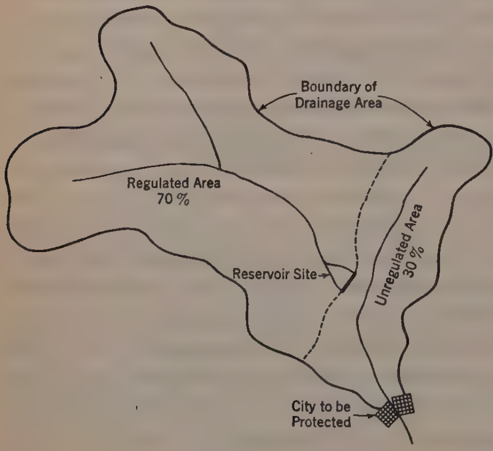

FIG. 12.

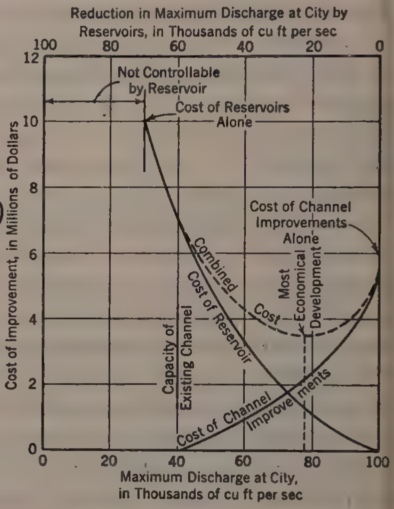

FIG. 13.

If reservoirs alone are used to hold the peak flood within the capacity of the existing channel, an expenditure of $10 000 000 may be required. If, on the other hand, the reservoir is not developed and flood protection is obtained solely by improvements to the channel, construction of the levees, etc., the cost to care for the maximum flood may be only $6 000 000. On the face of this, it would indicate that the use of channel improvements alone might be the most economical solution.

However, there are possibilities of combined development which will provide a more economical arrangement. Suppose, for instance, that the channel is improved to increase its carrying capacity to 50 000 cu ft per sec of the maximum flood peak for which the system is being designed and that the reservoir is developed to limit the flood flow in the channel at the city to 50 000 cu ft per sec, or 50% of the maximum flood. As 30% of the drainage area is unregulated, this will mean that the reservoir must be capable of reducing the flow from the area controlled to 20 000 cu ft per sec, thus reducing the discharge at the city to 50 000 cu ft per sec. With such a development the costs would be as follows (taken from curves of Fig. 13):

Cost of reservoir	$4 950 000
Cost of channel improvements	450 000
Total	$5 400 000

This represents a saving of $600 000 over protection by channel improvements alone.

A study of further combinations of reservoir development and channel improvements (made from the curves of Fig. 13) indicates that a minimum cost of flood control will be obtained with a channel capacity of 78% of the maximum flood peak, or 78 000 cu ft per sec, and a reservoir capacity sufficient to reduce the flow at the city to 78 000 cu ft per sec. Under this condition the reservoir must reduce the flow from the regulated area from 70 000 cu ft per sec to 48 000 cu ft per sec, or by 22 000 cu ft per sec. With this arrangement the costs are as follows:

Cost of reservoir	$1 300 000
Cost of channel improvement	2 200 000
Total	$3 500 000

This simple example is illustrative of the results that may be obtained by an intelligent combination of reservoirs and channel improvements. Naturally, the usual flood-control project will not be as simple as the hypothetical one considered herein, but the method illustrated is capable of application. Such methods were developed and used in the design of the Miami Conservancy District[8] which, of course, presented a much more complicated plan.

Reservoir and channel improvements both have their places in flood-control work; both have their advantages; and both have their limitations. The design of any given flood-control project should not be concerned solely with the relative merits of reservoirs alone or channel improvements alone, but should be concerned with obtaining the most economical plan, which may often be a combination of reservoirs and channel improvements.

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[8] "Hydraulics of Miami Flood Control Project", by Sherman M. Wood, Technical Repts., Miami Conservancy Dist., Pt. 7, Chapter 12.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

FLOW OF WATER AROUND BENDS IN PIPES

Discussion

BY MESSRS. D. BENJAMEN GUMENSKY, WALLACE M. LANSFORD, AND F. T. MAVIS.

D. BENJAMEN GUMENSKY,[7] ASSOC. M. AM. SOC. C. E. (by letter).[7a]—The explanation of the phenomena of flow around bends, contained in this paper, is very elucidating. Particularly clear and useful are the sections, "Velocity Conditions Within the Bend" and "Pressure Changes Within the Bend." The section on "Loss of Head in Bends" is of extreme interest to the designer of large pipes where the cost of head is high. In such cases the loss of head due to bends may have considerable influence on the selection of the most economical size for the pipe.

The loss of head due to bends was a subject of study in connection with the design of large siphons in the Colorado River Aqueduct. This aqueduct, now under construction (1934) by The Metropolitan Water Distric of Southern California, has in its main part 150 siphons, aggregating 27 miles in length. In these siphons, head losses due to bends are computed by the empirical formula:

$$h_b = K \left(\frac{V^2}{2\,g}\right) \quad \text{.............................(4)}$$

in which, h_b is the loss in one bend; V, the mean velocity in the pipe; and g, the acceleration due to gravity; and,

$$K = C \sqrt{\frac{\Delta}{90}} \quad \text{.............................(5)}$$

in which, C is a coefficient depending on the ratio of pipe diameter to radius of bend; and Δ is the deflection angle, in degrees.

Equation (4) was devised several years ago by Julian Hinds, M. Am. Soc. C. E., in an arbitrary manner, after a study of the extremely discordant literature on the subject available at that time.

NOTE.—This paper by David L. Yarnell, M. Am. Soc. C. E., and the late Floyd A. Nagler, M. Am. Soc. C. E., was presented at the Joint Meeting of the Power Division of the Society and the Hydraulics Division of the American Society of Mechanical Engineers at the Annual Convention, Chicago, Ill., on June 29, 1933, and published in August, 1934, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[7] Asst. Engr., The Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

[7a] Received by the Secretary September 24, 1934.

Although the total head allowed in siphon bends on the aqueduct is appreciable, the effect of bends is small in comparison with other factors, and it was considered safe to proceed with Equation (4) without attempting to substantiate it experimentally. However, the writer and others were interested in the problem and attempted to set up a small-scale experiment on their own initiative. R. R. Proctor, Assoc. M. Am. Soc. C. E., and members of his staff, made water and facilities available and assisted with the experiment. It is described herein in order to supply additional experimental data, and in the hope that the several points of discussion will merit Mr. Yarnell's attention in the closing discussion.

The experiment consisted in measuring the bend losses in a 2 by 2-in. square pipe for different angles of bend and different radii of curvature. Measurements were made on bends of 180°, 135°, 90°, 45°, and 20° for radii of curvature of 20, 8, and 4 in. Velocities varied from 5 to 16 ft per sec.

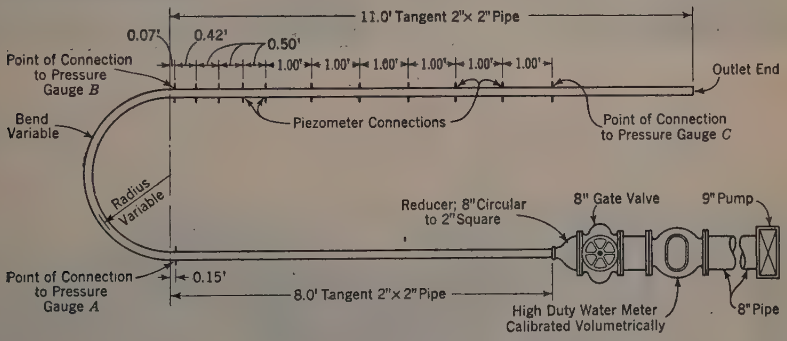
11.0' Tangent 2"× 2" Pipe
0.07'
0.42'
0.50'
1.00'
1.00'
1.00'
1.00'
1.00'
1.00'
Point of Connection to Pressure Gauge B
Outlet End
Bend Variable
Piezometer Connections
Point of Connection to Pressure Gauge C
Radius Variable
Reducer; 8" Circular to 2" Square
8" Gate Valve
9" Pump
Point of Connection to Pressure Gauge A
0.15'
8.0' Tangent 2"× 2" Pipe
High Duty Water Meter Calibrated Volumetrically
8" Pipe

FIG. 10.—EXPERIMENTAL SET-UP

The experimental set-up is shown in Figs. 10, 11, and 12. The water was pumped from a well through a calibrated meter and a reducer into an 8-ft tangent of 2 by 2-in. square pipe; then it was passed through a bend, an 11-ft tangent, and discharged into the air. The outlet end of the straight tangent had to be constricted in order to build up back pressure. The same two tangents were used for all set-ups; and each bend was carefully fitted and aligned between the tangents. The connections were made by soldering the seams from the outside and being careful that no solder would project into the section of the pipe and form an obstruction.

With a differential pressure gauge the loss of head was measured from a point 2 in. “‘up stream” from Bend (*A*) to a point 1 in. “down stream” from the end of Bend (*B*), and from the latter point to a point 8 ft farther along the tangent to Point (*C*). Then, as a check on the two measurements, an over-all loss was measured from the point above the bend to the point near the outlet end, Gauge (*A*) to Gauge (*C*). This measured total loss of head was compared with the experimentally determined loss of head in the identical lengths of straight pipe for the same velocity. The difference was considered to be the loss due to the bend.

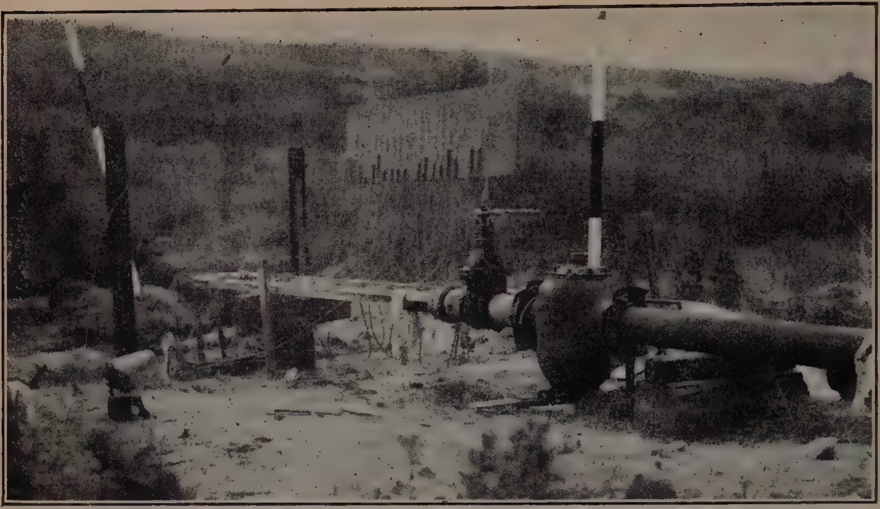

FIG. 11.—SET-UP FOR 180 DEGREE BEND

The results reduced to coefficients of velocity head in Equation (4) are shown in Fig. 13. Although the work was done with rather crude equipment the results are reasonably consistent. The general trend of variation and the actual values of losses seem to coincide quite definitely with those reported by the authors. The solid, horizontal lines denoting values of $C \sqrt{\frac{\Delta}{90}}$ were defined by the results of the test. The curve for K by Equation (5) (with $C = 0.25$) is that used by The Metropolitan Water District of Southern California.

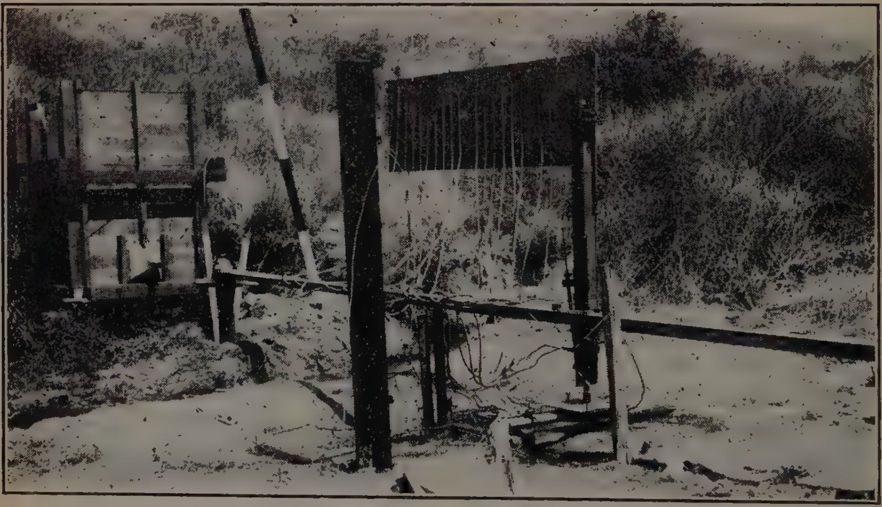

FIG. 12.—SET-UP FOR 10 DEGREE BEND

All the observed losses are smaller than those computed from Equation (4). Ignoring the observations for curves of 180° deflection, the average of all readings falls almost exactly on the line:

$$K = 0.16 \left(\frac{\Delta}{90} \right)^{0.845} \quad \text{.......................} (6)$$

The coefficients of bend loss for 180° fall out of the line with others and proportionately are much smaller. No satisfactory explanation of this deviation can be offered. No regular variation of the value of K with radius was discovered. If the observations are segregated as to velocity, it is found that the value of K increases with increasing velocity. This would indicate that the bend loss is proportional to a power of V greater than two, which is contrary to Conclusion (7) of the paper. The character of the experiment and the results obtained do not warrant an attempt to determine the exact value of this power.

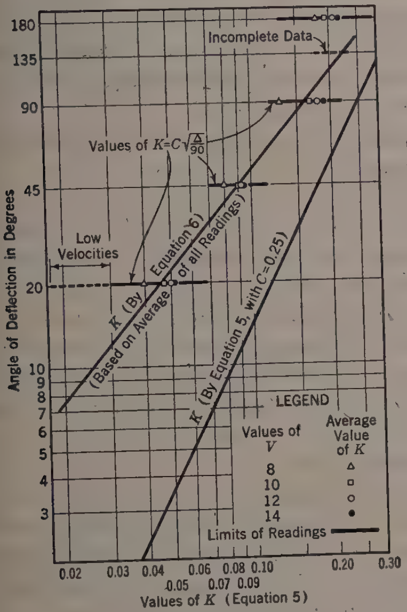

FIG. 13

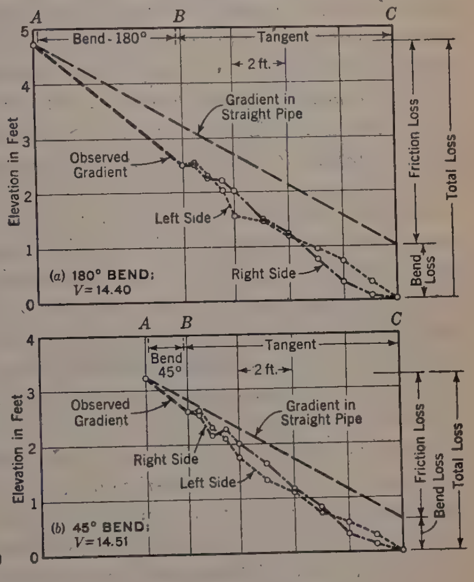

FIG. 14.—TYPICAL BEND LOSS MEASUREMENTS

Actual observations have shown that the pressure on both sides of the straight pipe below the bend varied, changing from the position above the average value to that below it, as if the water were bouncing from one side of the pipe to the other. This was disclosed by observation on piezometers connected to both sides of the tangent below the curve, which can be seen on Figs. 11 and 12. Fig. 14 shows the typical measurements and the losses. The curves shown are not smoothed out, but are drawn through points of individual measurements.

The loss of head due to bend was measured not entirely in the bend, but also in the straight pipe following. This is well shown on Fig. 14. Similar results were found by Mr. A. W. Brightmore[8] in his experiments on elbows and bends in 3-in. and 4-in. pipe. Examining the authors' Fig. 8, it is readily seen that conditions were similar to those described herein. The energy gradient in a tangent following the bend is steeper than that for the straight pipe and does not become parallel to it for a distance varying between 5 and 15 ft from the end of the bend.

It is a regrettable fact that all the observations made thus far have been on pipes of small diameters. Application of principles of similitude to the existing experimental data in order to determine the probable losses in large pipes would be uncertain and speculative, although in actual practice engineers are forced to assign definite values to bend losses in large pipes.

It is sincerely hoped that in the near future some one will be able to make measurements of bend losses in large pipes.

WALLACE M. LANSFORD,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—The suggestion of the authors to use pipe bends as flow meters is a valuable one, although they are not the first[10] to propose such a possibility. The writer first heard of this method of measuring the flow of water in the discussion of this paper by the late Professor Nagler following its oral presentation.[1] In the summer of 1933 the writer made a series of tests which seem to verify two conclusions reached by the authors, first, that an elbow in a pipe line could be used as a flow meter; and, second, that where great accuracy is required, each elbow should be calibrated.

Since relatively few experimental data are given in the paper, the writer's experiments will be stated in some detail. Tests were made on three cast-iron, flanged elbows located in a pipe line in the Hydraulics Laboratory of the University of Illinois, under the direction of M. L. Enger, M. Am. Soc. C. E. The radius of the inside bend of the elbows and the nominal diameter of both pipe and elbows were 4 in. The length of straight pipe preceding each elbow was 150 ft 2 in.; 8 ft 6 in.; and 128 ft, respectively. The pressure connections, ¼ in. in diameter, were made on both the inside and outside bends of each elbow at points approximately 45° from each flange in the plane of symmetry of the bend. An individual differential gauge was used for each elbow. The fluid (carbon tetrachloride and gasoline dyed red) used in all three gauges was taken from one mixture, thereby assuring identical coefficients. The velocity in the pipe was controlled by a valve at the outlet end. The discharge was measured by letting the water flow into a calibrated pit tank which had a cross-sectional area of 28.20 sq ft, and noting the rise of the water and the elapsed time of each run. The water could be turned quickly into the pit tank or diverted from it by means of a swinging pipe.

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[8] *Minutes of Proceedings*, Inst. C. E., Vol. 169, pp. 315 *et seq.*

[9] Assoc., Theoretical and Applied Mechanics, Unv. of Illinois, Urbana, Ill.

[9a] Received by the Secretary October 8, 1934.

[10] "New Method of Water Measurement by Use of Elbows in a Pipe Line," Gaskell S. Jacobs and Francis A. Sooy, *Journal of Electricity, Power, and Gas*, July 22, 1911, Vol. 27; "Die Turbinen für Wasserkraftbetrieb," von A. Pfaar, 1912. (He gives the following references: Isaachsen in *Zivilingenieur*, 1886, p. 338, and 1896, p. 352; Kankelwitz (Stuttgart), about 1870); "A Flow Metering Apparatus," by A. M. Levin, *Journal*, A. S. M. E., September, 1914; "The Hyperbo-Electric Flow Meter," *Power*, June 26, 1923; and "The Flow in Curved Pipes and Its Stability," by Otogoro Miyagi, Vol. XI, No. 1, Technology Repts., Tohoku Imperial Univ.

[1] At the Joint Meeting of the Power Division of the Society and the Hydraulics Division of the American Society of Mechanical Engineers at the Annual Convention, Chicago, Ill., on June 29, 1933.

During any one run all three gauges were read when the water was flowing into the pit tank, thus assuring the same discharge through each elbow for any one set of readings. Time was measured by a stop-watch. Water was taken from a stand-pipe, 6 ft in diameter and 65 ft high, near the top of which was an overflow weir approximately 15 ft long. Water flowed over this weir continuously, thereby maintaining constant pressure throughout all the tests. Data were taken in several runs with velocities varying from 0.77 ft per sec to 5.72 ft per sec.

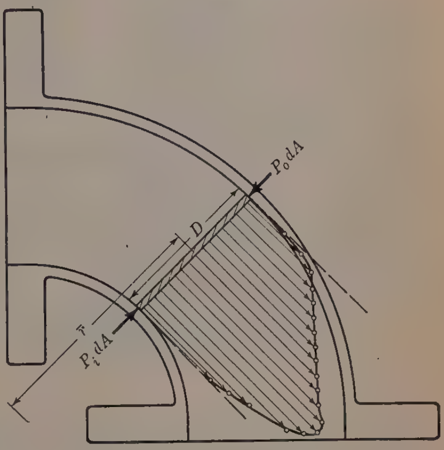

FIG. 15.—SECTION THROUGH CENTER OF ELBOW SHOWING VELOCITY DISTRIBUTION

Using a small prism of water of length, L, and of cross-sectional area, dA, as shown in Fig. 15:

$$(p_o - p_i)\ dA = \left(D\,\frac{V^2}{\bar{r}}\right)\frac{w}{g}\ dA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and,

$$h = \left(\frac{p_o}{w} - \frac{p_i}{w}\right) = \left(\frac{2\,D}{\bar{r}}\right)\frac{V^2}{2\,g} = k\,\frac{V^2}{2\,g} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

in which, V = mean velocity of water in pipe, in feet per second; p_i = unit intensity of pressure at inside of bend, in pounds per square foot; p_o = unit intensity of pressure at outside of bend, in pounds per square foot; $\bar{r}$ = radius of the mass center of the prism, in feet; and D = diameter of elbow, in feet.

Equation (8) shows that the difference in pressure head between the outside and inside of an elbow through which water is flowing, is a constant times the velocity head (based on the average velocity), provided the value of $\bar{r}$ does not change for different rates of flow. Ordinarily, the radius, $\bar{r}$, is less than the radius of the center line of the elbow, because the velocity of flow is highest near the inside of the bend. However, with abnormal distribution of velocity at the entrance of the elbow, the radius, $\bar{r}$, might be increased or decreased.

The curves in Fig. 16 show the relation between c_k and the average velocity in the pipe. It can be seen from these curves that c_k is a constant for each elbow when the velocity of the water in the pipe is approximately 1.5 ft per sec, or greater. This agrees with the conclusion of the authors that the elbow is valuable as a meter, and that each elbow should be calibrated for accurate use. The more constant value of c_k for each bend was, as follows (see Fig. 16):

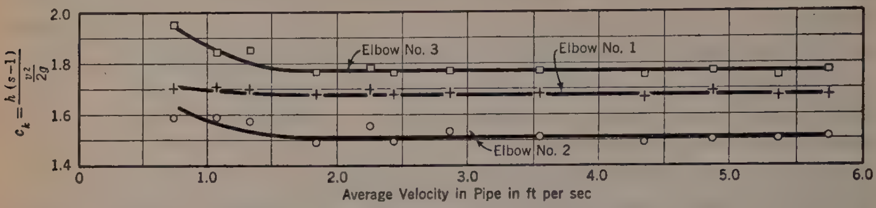

FIG. 16.—RELATION BETWEEN c_k AND VELOCITY IN PIPE FOR THREE 4-INCH ELBOWS (RADIUS OF INSIDE BEND, 4 INCHES)

Elbow No.	Value of c_k
1	1.68
2	1.51
3	1.77

Later, all elbows were interchanged in all positions in the pipe line, but the direction of flow through them was not changed. Values of c_k for the elbows, when in the latter positions, deviated about 3% from those shown in Fig. 16. This difference could be due to the effect of changing gaskets, or to the eccentricity of the elbows caused by a large clearance of the bolts in the holes through the flanges. The length of straight pipe preceding each position of the elbows seemed to be sufficient not to affect the value of c_k.

From Equation (8) the theoretical value of c_k for the elbows tested would be $c_k = \frac{2D}{\bar{r}} = 1.33$, if $\bar{r}$ is taken as the radius to the center of the elbow. In order to investigate the discrepancy between the actual and theoretical values of c_k, a Pitot tube traverse was made of Elbow No. 2 when in the southeast position where it was preceded by 150 ft 2 in. of straight pipe. The measured velocity curve is shown in Fig. 15. This curve is quite different from that assumed in the theoretical analysis. A rough calculation of $\bar{r}$ for this velocity distribution, assuming the water to flow in stream lines, shows it to be about 5.72 in. instead of 6 in. Therefore, c_k, would be $\frac{2\,D}{5.72} = 1.40$. It will be noted that a small variation in $\bar{r}$ changes the value of c_k materially; thus, when $\bar{r}$ is 5.33 in., $c_k = 1.50$. This would probably account for some of the difference between the actual and theoretical values of c_k.

On the basis of the information obtained from the tests herein reported, a useful device known as a flow indicator has been built and is being used successfully in the Hydraulics Laboratory of the University of Illinois. The centrifugal pumps that supply the water are on the basement floor, approximately 5 ft above the level of the water in the supply channel. The control switches for the motors that drive the pumps are on the floor above. When a pump is started it is convenient and important to know whether or not it is actually delivering water. The device is shown in Figs. 17 and 18. It consists essentially of two closed steel cylinders approximately $1\frac{3}{4}$ in. in inside diameter and 3 in. long, mounted end-wise on a steel bar and balanced on the bolt at mid-span as a fulcrum. The cylinders, about one-half full of mercury, are directly connected beneath by $\frac{1}{8}$-in. pipe. The $\frac{3}{16}$-in. copper tube that connects the left cylinder to the outside bend of the elbow and the right cylinder to the inside is shown in Fig. 18. Water flowing through the elbow produces a high pressure at the outside of the bend and a low pressure at the inside, which causes the mercury to flow from the left cylinder to the right cylinder. The unbalanced weight causes the right end of the indicator to deflect and to make contact with an electric terminal. This closes an electric circuit which, in turn, lights a small electric light above the control switch on the first floor. When the pump is stopped the mercury flows back into the left cylinder, thus equalizing the weight and breaking the electric circuit. Two of these indicators are used on two pumps having a discharge capacity of 1 000 gal per min, each, and two on two pumps having a discharge capacity of 2 000 gal per min, each. All the indicators were installed in January, 1934, and have given good service.

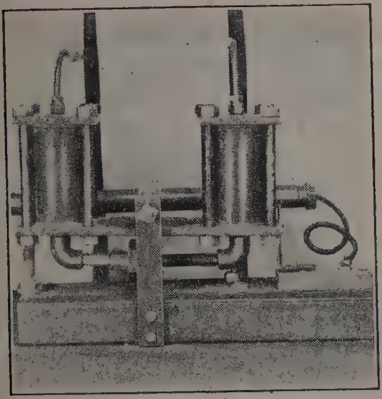

FIG. 17.—THE FLOW INDICATOR

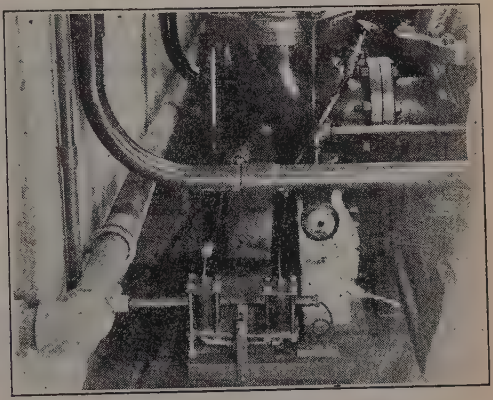

FIG. 18.—THE FLOW INDICATOR INSTALLED

F. T. MAVIS,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—A valuable summary of a long series of painstaking tests is presented in this paper. The authors deserve much credit for reviving the use of a bend as a flow meter and for their leadership in this particular research, which has also been included in the programs of other laboratories in the United States.

Attention should be called to the work of other investigators who have made significant contributions to the investigations of the hydraulics of bends in pipes. The pioneer work of Weisbach[12] in Germany about the middle of the Nineteenth Century is often referred to in current hydraulic literature. Twenty-five years or more ago, Alexander[13] and Brightmore[14] were conducting tests in England, and Williams, Hubbell, and Fenkell,[15] and Schoder[16] were making tests in this country. Among the current investigators might be mentioned the names of Hoffmann[17] and Schubart[18] in Munich, Nippert,[19] in Danzig, and Giesecke,[20] and the Staff of the National Hydraulics Laboratory in the United States. Most of these experiments were conducted on pipes of circular cross-section, although some tests were made on pipe of square or rectangular section. Doubtless, there are many other published and unpublished investigations which could be brought together, re-appraised, and correlated to make an exceedingly valuable reference work on the subject of flow of water around bends.

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[11] Associate Director in Chg. of Laboratory, Iowa Inst. of Hydraulics Research; and Associate Prof., Dept of Mechanics and Hydraulics, The State Univ. of Iowa, Iowa City, Iowa.

[11a] Received by the Secretary October 15, 1934.

[12] "Lehrbuch der Ingenieur und Maschinen Mechanik," von J. Weisbach, p. 439, Leipzig, 1845.

Many tests have been made at the University of Illinois by M. L. Enger, M. Am. Soc. C. E., and W. M. Lansford, Assoc. M. Am. Soc. C. E., to study the properties of bends as flow meters. In 1916, the idea of tapping the inside and outside of a bend and using the pressure difference in these two points as a means of determining the flow of water through the bend, was credited to Mr. Buckner Speed by the late J. C. Trautwine, Jr.,[21] Affiliate, Am. Soc. C. E. Mr. Trautwine also referred to tests reported by Messrs. G. S. Jacobs and F. A. Sooy,[22] which led to an equation expressing the relation between the velocity in the pipe, V, the pressure difference, h, in feet of water, the radius of the bend, r, and the diameter of the pipe, D, as follows:

$$V = c\sqrt[u]{h\frac{r}{D}} \qquad (9)$$

in which, $c = 5.6$ and $u = 1.9$. Other data and calibration tests of a similar flow-metering apparatus were reported in 1914, by Mr. A. M. Levin.[23]

The problem of the hydraulics of bends has been investigated in some detail from an analytical basis. zur Nedden has presented certain aspects of the problem which merit the mention of his paper[24] in particular. He calls attention to the fact that a bend may perform quite differently, depending on whether the flow of the fluid through it is laminar or turbulent. He cites specifically the case of a bend of rectangular cross-section the inner and outer curves of which were hyperbolic, as causing a lower head loss than a circular bend as long as the flow remained laminar; as soon as the flow became turbulent, the hyperbolic bend caused a greater head loss than the circular bend. This behavior would be apparent, of course, if the coefficients of velocity head, representing the excess head loss in the bend, were plotted as a function of Reynolds' number.

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[13] "The Resistance Offered to the Flow of Water in Pipes by Bends and Elbows," by C. W. L. Alexander, *Minutes of Proceedings*, Inst. C. E., Vol. 159, 1904–05, Pt. I, pp. 341–364.

[14] "Loss of Pressure in Water Flowing Through Straight and Curved Pipes," by A. W. Brightmore, *Minutes of Proceedings*, Inst. C. E., Vol. 169, 1907, p. 315.

[15] "Experiments at Detroit, Michigan, on the Effect of Curvature on the Flow of Water in Pipes," by the late Gardner S. Williams, M. Am. Soc. C. E., and Clarence W. Hubbell and George H. Fenkell, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. XLVII (1902), pp. 1 to 369.

[16] "Curve Resistance in Water Pipes," by E. W. Schoder, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXII (1909), pp. 67 to 112.

[17] "Neue Untersuchungen über den Druckverlust in Rohrkrümmern," von A. Hoffmann, *Mitteilungen* des Hydr. Inst. der Techn. Hochschule, München, Heft 2, pp. 70–71, 1928; also, Heft 3, pp. 45 ff, 1929.

[18] "Der Energieverlust in Kniestüchen bei glatter und rauher Wandung," von W. Schubart, *Mitteilungen* des Hydr. Inst. der Techn. Hochschule, München, Heft 3, pp. 121–144, 1929. ("Energy Loss in Smoooth and Rough Bends and Curves in Pipe Lines," tr. by F. T. Mavis, Assoc. M. Am. Soc. C. E.)

[19] "Uber den Strömungsverlust in gekrümmten Kanälen," von H. Nippert, *Forschungsarbeiten*, Heft 32, 1929.

[20] "Friction of Water in Elbows," by F. E. Giesecke, M. Am. Soc C. E., *Transactions*, Am. Soc. of Heating and Ventilating Engrs., Vol. 32, 1926, pp. 303–314.

[21] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 911.

[22] *Journal of Electricity, Power, and Gas*, San Francisco, Calif., July 22, 1911.

[23] "Flow Metering Apparatus," by A. M. Levin, *Transactions*, A. S. M. E., Vol. 36, 1914, pp. 239–254.

To-day, it is generally recognized that in flowing around a bend under turbulent flow conditions, a liquid moves in a so-called "spiral" or "helical" path, and that the induced currents and attendant impact are largely responsible for excess losses of head in bends. In 1902, the late H. W. Brinckerhoff, M. Am. Soc. C. E., discussing the paper by Messrs. Williams, Hubbell, and Fenkell,[15] explained the fact that a bend offers greater resistance than an equal length of straight pipe on an hypothesis of spiral flow which he described in its essentials precisely as other writers have done. In their closing discussion, Messrs. Williams, Hubbell, and Fenkell replied to Mr. Brinckerhoff's comments as follows:

> "At one time, one of them [the writers] was disposed to explain certain phenomena on the theory of a spiral motion, but in spite of attempts to locate such motion, no direct experimental evidence of its existence has yet been obtained, and, though the writers consider that Mr. Brinckerhoff's theory is ingenious and possibly correct, they are not prepared to endorse it."

Transparent model bends, such as those used by the authors and C. A. Mockmore,[25] M. Am. Soc. C. E., permit the observer to see this spiral or helical flow. These transparent models have done much to create a better understanding and a more intimate knowledge of the phenomena of hydraulics. To-day, this spiral flow in bends can be observed; three decades ago, it was an hypothesis which eminent hydraulic engineers were then "not prepared to endorse."

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[24] "Induced Currents in Fluids," by F. zur Nedden, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 844–913.

[25] "Flow in Bends of Quarter-Turn Draft-Tubes," by C. A. Mockmore (Abstract), *Civil Engineering*, September, 1934, p. 460.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

STREET THOROUGHFARES
A SYMPOSIUM

Discussion

By DONALD M. BAKER, M. AM. SOC. C. E.

DONALD M. BAKER,[13] M. AM. SOC. C. E. (by letter).[13a]—Just as many thousand miles of steam railroads were built before the economic principles of railroad construction were evolved and put in practice, so thousands of miles of highways were constructed and improved before such principles underlying their development were formulated and used. The various papers in this Symposium well set forth the principles underlying the use, design, and location of street thoroughfares. The definitions contained therein will be of material assistance to those engaged in solving traffic and vehicular transportation problems.

Of these principles, now fairly well recognized, least is known concerning those relating to highway financing, particularly in urban areas. Current practice is, through gasoline and motor-vehicle taxes, to place the major portion of the cost of constructing and maintaining highways located outside urban areas upon the users of such highways, while in urban areas such costs are borne to a large extent by property served, or assumed to be served.

The reasons leading to the development of such practices are of more practical than scientific soundness. It was soon found that values of suburban and rural property were insufficient to support the costs involved, and that the benefits accruing to such property were small in proportion to those accruing to the users of such highways. Therefore, if such highways were to be constructed and maintained to proper standards and in sufficient number to satisfy the growing demand, their costs would have to be borne principally by those using them.

In the case of highways in urban areas, a different situation existed. Urban property values were many times higher than those in outside areas, and such properties at the outset were considered well able to bear construction and maintenance costs. During the decade, 1920–1930, which saw rising

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NOTE.—The Symposium on Street Thoroughfares was published in August, 1934, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1934, by H. Shifrin, M. Am. Soc. C. E.

[13] Cons. Engr., Los Angeles, Calif.

[13a] Received by the Secretary October 15, 1934

traffic congestion becoming a serious menace and, during which, many street thoroughfares were developed, rising prices and the general real estate "boom" throughout the country caused, at the outset, an increase in property values in areas served by these new thoroughfares, out of proportion to the general rise in property values. A change in the traveling habits of the public, brought about by the increasing use of the motor vehicle, likewise contributed to this rise.

Sales of property were made at large profit, more thoroughfares were developed, and more profitable sales were made. No objection was raised by property owners to this method of financing thoroughfares because of these increased values or because such costs were largely paid by subsequent purchasers of their property. Gradually, however, the operation of economic laws came into play. The margin of profit between benefits, measured in terms of enhanced property values, and costs charged against the property which paid the costs were reduced and in some cases disappeared. This condition was greatly aggravated by the deflation in property values which took place in the period following 1929.

The need for additional street space persisted. Since local tax revenues, reduced through deflated property values and burdened by unemployment relief measures, were not sufficient to carry the cost of providing this additional street space, a demand arose for a greater allocation of gasoline and motor-vehicle tax revenues to urban highway construction and maintenance.

The problem of highway financing has four phases: (1) The determination of the different interests benefited by the construction and continued operation of the highway; (2) the evaluation of the benefits accruing to each of the interests thus benefited; (3) the allocation of construction and maintenance costs among the benefited interests in an equitable manner; and (4) the development of a method of collecting such costs from such interests.

The following principles should be observed as controlling in any policy of highway financing, whether it apply to urban or outside highways: (*a*) The benefits received may be direct or indirect; (*b*) the benefits must ultimately be expressed in terms of dollars and cents; (*c*) the costs should be allocated in proportion to, but must not exceed, the benefits derived; (*d*) the costs should be within the ability of the benefited interests to pay; and, (*e*) the method of collecting such costs must be simple and practical, yet sufficiently equitable as to meet with popular approval and support.

The problem does not admit of an exact solution, but an approximate one may be developed. The interests benefited fall into the following classifications: (1) Users of the highways, principally passenger and commercial motor vehicles; (2) property served by the highways, including (*a*) property abutting the highway; (*b*) property in an area adjacent to the highway, but not abutting thereon; and (*c*) a larger area within the community which is indirectly served and, therefore, benefited.

The gasoline and motor-vehicle tax appears to be generally accepted as a satisfactory method of financing highways outside urban areas on account of its approximate equity and it ease of collection. Unless large inroads are made upon these revenues, for urban highways and for other purposes, this method will probably continue to prove satisfactory for future needs. Because of changed conditions, a new and more equitable method of financing urban highways, particularly street thoroughfares, appears to be necessary. Determination of the interests benefited, and the evaluation of benefits accruing must be made in a more scientific manner. The development and application of sound principles for financing these thoroughfares is the crux of the entire problem. The writer agrees in the recommendation of the Committee that "a new type of street for automobiles is now needed in some cases, and will soon be needed generally", but does not believe that such new type of street can ever be achieved under present or near future economic conditions, until different methods of financing have been developed.

The writer does not agree with the principle presented that right-of-way and construction costs should be considered separately in assessing benefits and damages. This principle appears in the report to be based upon the idea that, since inadequate rights of way were dedicated by the original owners of property, their successors in interest should bear the burden of rectifying this mistake. Such inadequate rights of way were dedicated because no one asked for adequate rights of way at the time of dedication, in the case of the older streets, and probably public authorities at the time had no conception of present traffic congestion, or could have asked or intelligently requested, such right of way had they so desired.

During the years, 1927 to 1930, the Los Angeles (Calif.) Board of Planning Commissioners reviewed approximately a thousand subdivisions. Practically every subdivider was quite willing to dedicate street space of ample width and in proper location when asked to do so. To place the burden of rectifying past mistakes of previous owners upon existing owners, when such past mistakes were unconsciously made, appears to be an unfair principle. If a fair and equitable method of ascertaining benefited interests and of evaluating benefits accruing is developed, there will be no need of considering, separately, right-of-way and construction costs.